PROBLEMATIC TOPICS IN HIGH SCHOOL MATHEMATICS REQUIRING UNIVERSITY MATHEMATICS: EXPLORING THE NSC MATHEMATICS EXAMINATION

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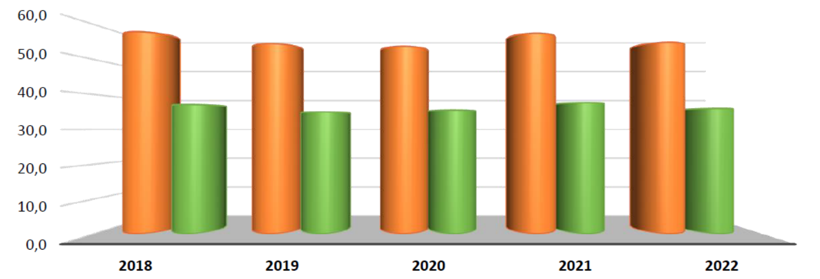
This article discusses the challenges high school mathematics learners face when transitioning to university mathematics, specifically with calculus and trigonometry. The study analysed the South African high school mathematics curriculum and the mathematics NSC examiners' reports, revealing a significant gap between high school and first-year university mathematics major courses. The researchers recommend an alteration of the South African high school curriculum to allow students to specialise in limited subjects directly linked to their future studies, specifically in calculus and trigonometry. This specialisation would allow for the introduction and deeper exploration of topics such as integration, complex numbers, differential equations, vector analysis, and matrices, which are currently missing from the mainstream high school mathematics curriculum. The study aims to inform support programmes such as tutorials and bridging courses offered at universities to be more effective in addressing the school-to-university transition problem for mathematics and science students.

INTRODUCTION

Mathematics is a critical subject in South Africa and globally, and many high school learners aspire to pursue mathematics-related courses in universities. However, transitioning from high school mathematics to university mathematics can be challenging. In this article, we will explore the challenges that high school mathematics learners experience when transitioning to university mathematics in general and focus on the problematic topics – calculus and trigonometry. The findings from this disjuncture could inform the support programmes like tutorials and bridging courses offered at universities to be more effective.

Several curriculum policy reforms have been implemented in South Africa since its independence in 1994. These include Curriculum 2005 (C2005) introduced in 1997, Revised National Curriculum Statement (RNCS) in 2002, National Curriculum Statement (NCS) in 2007, and Curriculum and Assessment Policy Statement (CAPS) launched in 2012 (Adu & Ngibe, 2014). We appreciate the efforts made hitherto in trying to address historical imbalances and increase learning opportunities for black students. However, the school-to-university transition problem for mathematics and science students remains unresolved and the gap appears to be getting wider (McDougall, 2015). The present study delineates and unravels the nature of the gap between high school and first-year university mathematics.

The class of 2022 entered the formal schooling system in Grade 1 in January 2011 and became the largest cohort to sit for the National Senior Certificate (NSC) examinations. The number of candidates who sat for the mathematics examinations in 2022 increased substantially by 10,591 candidates (Department of Basic Education, 2022a) While the table below indicates a decline in the pass rate over the period 2018-2020, improvement was evident in 2021.



	2018	2019	2020	2021	2022
% achieved at 30% and above	58,0	54,6	53,8	57,6	55,0
% achieved at 40% and above	37,1	35,0	35,6	37,6	36,0

Fig. 1: NSC results for the past five years (Department of Basic Education, 2022b)

Further analysis shows that candidates who passed at 30% (Level 2) declined slightly from 57,6% in 2021 to 55,0% in 2022. There was a corresponding decline in the pass rate at 40% (Level 3) over the past two years from 37,6% to 36,0%. There was also a marginal decline in the percentage of distinctions over 80% (Level 7), which decreased from 2,9% in 2021 to 2,7% in 2022. In view of the disruption to academic programmes in 2020 and 2021, the results achieved by this cohort are commendable. Strategic intervention programmes at all levels (National, Provincial, Districts and Schools) ensured that learners were adequately prepared. The diligence and perseverance of the above-average candidates also contributed to the overall performance (Department of Basic Education, 2022b).

The class of 2022 has had the advantage of being the ninth cohort to be exposed to examinations based on the Curriculum Assessment Policy Statements (CAPS). However, there is still a curriculum gap between high school and university mathematics which makes the transition from high school to university difficult, resulting in high failure and attrition rates and low throughputs at universities (Reddy et al., 2014). If many students take longer to graduate, it means we are spending more to produce one graduate, and there is no room to admit new students into mathematics programmes. Worse still when students drop out of university, the money invested in the students during their time at university will never be recovered. In addition, students who drop out of university are a burden to society since they are unemployable. Low throughput rates suggest that the country will continue to have a scarcity of qualified personnel in the critical skills areas of the economy.

While we appreciate the efforts being made by universities to support students who come to university with learning deficits in mathematics by offering extra tuition and allowing them to register for extended programmes, the responsibility to curb the crisis cannot be left to universities alone. A concerted effort from both institutions, high school and university, is required to bridge the gap and make the transition from school to university smoother for our future mathematicians, engineers and scientists. This study looks at a five-year period, from 2018 to 2022, with a special focus on common errors, misinterpretations and misconceptions identified in the calculus and trigonometry questions of the NSC mathematics examination. The results of the study could help curriculum developers to align the content and teaching of these two topics in high school and university. We, therefore, attempt to answer the following question: *What can be done to improve and align the content and teaching of calculus and trigonometry, to ease the transition to first-year university mathematics?*

LITERATURE REVIEW

A study conducted by the Council for Higher Education and the South African Institute of Physics showed that the current high school mathematics curriculum is failing university entrants (Reddy et al., 2014). This indicates that there is a discrepancy between the high school and university mathematics curricula that needs to be addressed. To gain a better understanding of the performance of candidates and identify weaknesses in the teaching and learning of mathematics at high schools, it is important to analyse reports published from 2015 to 2021. The 2022 NSC diagnostic report highlights the areas of weakness in the mathematics examination and outlines remedial measures that should be adopted at the school level to improve performance. This report provides valuable information to teachers, subject advisors, curriculum planners, and curriculum implementers (Department of Education, 2022b).

At the university level, several studies reported on the difficulty that students face in transitioning from high school to university mathematics. Hourigan and O'Donoghue (2007) found that students' inability to cope with the transition is due to the big difference between the nature of mathematics experience at pre-university level and that at university in mathematics-intensive courses. Other researchers such as Craig (2007) and Hoyles et al. (2001) have reported on the differences between university expectations and the wide spectrum of mathematical abilities of new students. The changing school curricula have been identified as one of the reasons for these differences (de La Paz, 2005; Hoyles et al., 2001). Clark and Lovric (2008) highlighted the transition to mathematics in higher education as a 'rite of passage'. There is a clear need to investigate the deficiencies in the current high school mathematics curriculum and the differences between high school and university mathematics curricula to improve the transition of students from high school to university mathematics.

Analysing reports on candidates' performance and identifying areas of weakness is a crucial step in this process.

THEORETICAL FRAMEWORK

The present study is rooted in the needs assessment theory (Long, 2005), which is an evolving theory that draws on contributions from researchers in various fields such as psychology, education, and management. In the context of education and curriculum development, Roger Kaufman (2020) has made significant contributions to the needs assessment theory. In South African mathematics education, there is an undeniable gap between the current high school mathematics curriculum and the university mathematics curriculum. As such, a needs assessment is warranted. Needs assessment is a systematic process that involves determining the nature of the discrepancy or gap between the current condition and the desired condition in order to improve performance and correct deficiencies. Needs assessment helps to clarify problems and identify appropriate remedial measures. By clearly identifying the problem, the study can channel resources towards developing and implementing a feasible solution. Furthermore, needs assessment provides concrete evidence that can be used to determine the most effective means of achieving the desired results. It guides policy and programme decisions, including the design, implementation, and evaluation of policies and programmes that will lead to the desired results.

Finally, the needs assessment approach adopted in this study will enable a thorough understanding of the gap between high school and university mathematics curricula in South Africa, as well as the identification of appropriate remedial measures to improve student performance in mathematics.

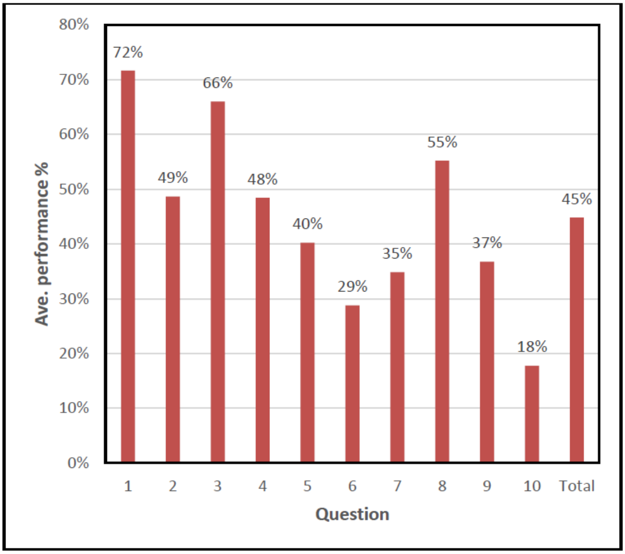
METHODOLOGY

This qualitative study will be conducted in two phases. In the first phase, a documentary analysis research method will be used to explore the performance of NSC candidates, and the concerns raised in the NSC examination report relating to calculus and trigonometry. This phase will also explore to what extent these topics are important for first-year university mathematics. The analysis will involve examining and interpreting existing documents, such as printed or electronic materials, with specific attention to identifying discrepancies and similarities between the mathematics content covered in FET (Grades 10-12) and first-year university mathematics.

In the second phase of the study, a survey will be conducted with lecturers and students from the 2023 intake. The survey will gather information about their experiences and opinions regarding the mathematics curriculum. This information will be used to guide further investigation, particularly in analysing the mathematics tests, in particular calculus and trigonometry questions, taken during the semester to gain a deeper understanding of the challenges faced by students and how they can be addressed.

PRELIMINARY FINDINGS AND DISCUSSION

Mathematics in the Further Education and Training (FET) phase covers ten content areas. These are Functions, Number Patterns, Sequences and Series, Finance, growth and decay, Algebra, Differential Calculus, Probability, Euclidean Geometry and Measurement, Analytical Geometry, Trigonometry, and Statistics (Department of Basic Education, 2011, p. 9). Each content area makes a contribution towards the acquisition of specific mathematical knowledge and skills. Differential Calculus is taught across the three grades in the FET Phase. If we look closely at the content specifications per content area, we see that in certain topics, the concepts and skills are similar across the grades. The difference lies in the level of difficulty. Thus, the curriculum is spiral in nature. It is envisaged that the FET mathematics curriculum will give learners "many opportunities to develop their mathematical reasoning and creative skills in preparation for more abstract mathematics in Higher/Tertiary Education institutions" (Department of Basic Education, 2011, p. 10). However, there are numerous reports indicating that learners are coming to university underprepared (Reddy et al., 2014). This comes in the wake of a high failure rate in first-year university mathematics among students enrolled in engineering and science programmes across universities in South Africa (Reddy et al., 2014). The following areas of concern were identified in the past five years' examination reports and are, once again, highlighted as concerns in the 2022 NSC examinations – calculus and trigonometry questions form part of the lowest-scoring sections.



Q	Topic(s)
1	Data Handling
2	Data Handling
3	Analytical Geometry
4	Analytical Geometry
5	Trigonometry
6	Trigonometry
7	Trigonometry
8	Euclidean Geometry
9	Euclidean Geometry
10	Euclidean Geometry

Fig. 2: Average performance per question in 2022 NSC mathematics paper 1 (Examiners' report, 2022)

Figure 2 indicates that Question 9 (calculus) appears to be the most challenging in Paper 1 of the 2022 NSC examinations. In Question 9 candidates were asked to calculate the minimum distance between a point and a parabola function (Figure 3).

QUESTION 9

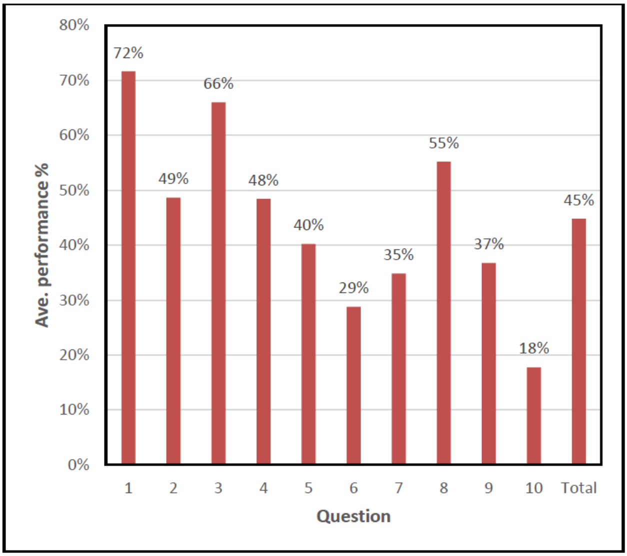
Given $f(x) = x^2$.

Determine the minimum distance between the point (10; 2) and a point on f

Fig. 3: Question 9 in the mathematics Paper 1(NSC examinations, 2022)

The diagnostic report indicates that "The vast majority of the candidates did not attempt this question because they were unable to determine the distance function from the given information" (Department of Education, 2022b: p.206). It also mentioned that some candidates used trial-and-error methods to obtain the minimum distance between the points. Learners appear to be dependent on the formulae being given when solving optimisation problems. It is advisable that learners interrogate the optimum function even when it is given in a question. This should help their conceptual development. It is also very important that teachers ensure that there is enough time for learners to understand the application of calculus fully, especially derivatives. Reading for understanding should be ongoing if learners are to improve their responses to word problems (problems with limited information).

In Figure 4, trigonometry is second from the bottom in terms of the performance of candidates in Paper 2.



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9	Euclidean Geometry
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Fig. 4: Average performance per question in 2022 NSC mathematics paper 2 (Examiners' report, 2022)

In Question 6 (trigonometry) the candidates were given two superimposed trigonometric functions (Figure 5). In order to answer this question, they were required to have conceptual knowledge about the characteristics of the tangent and sines functions.

QUESTION 6

In the diagram below, the graphs of $f(x) = \tan x$ and $g(x) = 2\sin 2x$ are drawn for the interval $x \in [-180° ; 180°]$ A$(60° ; k)$ and B are two points of intersection of f and g.

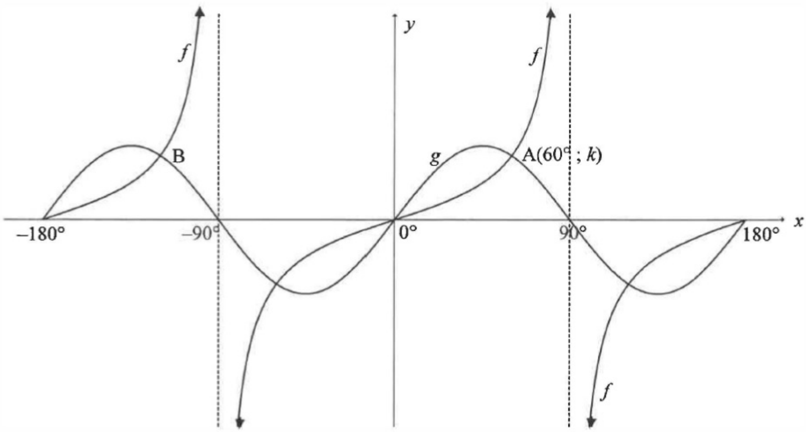

Fig. 5: Question 6 in the mathematics Paper 2 (NSC examinations, 2022)

The typical mistakes and misconceptions highlighted in the diagnostic report about this question were:

- Some candidates gave the domain, which is an interval, instead of the period, which is a single value.
- Although point B was to the left of the *y*-axis, some candidates wrote 120° instead of -120^0, and others also incorrectly provided −60° as an answer.
- Most candidates had little idea how the graphs could be used; they could not make association between sin(x), cos(x) and sin($2x$).

It is essential for learners to recall the trigonometry taught in Grades 10 and 11. Revision of this work must be ongoing. The same simplification skills used in algebra also apply to trigonometry and learners should regularly practise to remediate poor algebraic and manipulation skills. More emphasis should be placed on solving simple trigonometric equations, particularly those where a basic trigonometric equation has two solutions in the interval from 0° to 360°. The key to solving trigonometric equations lies in understanding in which quadrants a trigonometric function is positive or negative. Learners should be given exercises to practise solving complex trigonometric equations.

Overall, it was evident in candidates' responses that there is a general lack of reading and comprehension skills. It was noted that many candidates did not understand the vocabulary used in questions. A significant percentage of the 2022 candidates responded adequately to questions that required lower-order thinking skills. However, many candidates performed poorly in questions that demanded analytical, evaluative, and problem-solving skills.

In terms of interventions at high schools, teachers are urged to incorporate dialogical argumentation in the mathematics classroom; use technology, visual demonstrations, practical activities, and visualisation to improve learners' understanding of subject matter; make use of online learning platforms such as YouTube that offer visual presentations and explanations of challenging topics. These and other intervention strategies should be integrated to improve the conceptual understanding of mathematics topics, a concern pointed out in the NSC diagnostic report.

Mathematics challenges and support programmes at universities

The challenges of the school-to-university transition of mathematics mean that significant numbers of students have not mastered the basic mathematical skills required for first-year undergraduate mathematics modules (Reddy et al., 2014). However, it is not clear if students have difficulties with both the concepts and procedures involved (Caitriona et al., 2016). Several support programmes and initiatives implemented to address the gap between students' mathematical ability and the skills they require to succeed in first-year mathematics modules include diagnostic tests, tutorial programmes, bridging courses, extended programmes, etc. Caitriona et al. (2016) found that students are ignorant of their own fundamental problems in understanding mathematics. To address this need and help students develop understanding, lecturers should incorporate material such as interactive mathematical tasks specifically designed to improve understanding, in their syllabi. . Additionally, students need to be informed of their lack of understanding through formative assessments that give immediate feedback such as the use of audience response systems.

CONCLUSION AND RECOMMENDATIONS

This study highlights the significant gap between high school and university mathematics for science and engineering students in South Africa. While the current high school mathematics curriculum, based on CAPS, appears to cover a wide range of content areas, it lacks depth in critical content areas of mathematics and therefore does not adequately prepare students for university-level mathematics (Machisi, 2018). The findings of the first phase of this study indicate that there has been a relative decrease in the NSC candidates' performance over the past five years, with trigonometry and calculus constantly part of the worse performing subjects. Furthermore, this study also showed that many core aspects of algebra and calculus, such as complex numbers, vectors, matrices, proof by mathematical induction, integration, and differential equations, are excluded or covered inadequately in high school mathematics.

To narrow the gap between high school and university mathematics, we recommend reducing the number of FET subjects so that students specialise in fewer subjects, directly linked to their intended fields of study at university. Additionally, it is imperative to address the recurrent areas of weakness in teaching methodology and content coverage, particularly as teaching has been negatively affected by the COVID-19 pandemic.

Moving forward, the second phase of this study will examine the responses of lecturers and students to the gap between high school and university mathematics, in order to gain a better understanding of the experiences of those on the front lines of this issue. The ultimate goal of this study is to provide evidence-based recommendations for addressing the school-to-university transition problem in mathematics and science education, so that all students can have the opportunity to succeed in these critical fields.

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