International Journal of Applied Mathematics

Volume 35 No. 2 2022, 205-223
ISSN: 1311-1728 (printed version); ISSN: 1314-8060 (on-line version)
doi: http://dx.doi.org/10.12732/ijam.v35i2.1

NUMERICAL ANALYSIS ON UNSTEADY MHD FLOW AND HEAT TRANSFER OVER AN INCLINED STRETCHING SURFACE IN A POROUS MEDIUM WITH HEAT SOURCE AND VARIABLE MAGNETIC FIELD ANGLE

George Buzuzi[1], Gilbert Makanda[2 §]

[1]Department of Mathematics and Physics
Cape Peninsula University of Technology
P.O. Box 1906, Bellville 7535
SOUTH AFRICA

[2]Department of Mathematical and Physical Sciences
Central University of Technology, Free State
Private Bag X20539, Bloemfontein
SOUTH AFRICA

Abstract: Numerical analysis on unsteady magnetohydrodynamic flow and heat transfer over an inclined stretching channel in a porous medium with heat source in the presence of an inclined magnetic field is reported. The non-linear and coupled governing equations are solved by transforming them to ordinary differential equations by using similarity transformation and are then solved numerically by applying the successive linearization method. Convergence analysis of the method is carried out to show the robustness of the method. The effects of the various flow parameters on the velocity and temperature fields as well as the skin friction coefficient and the heat transfer coefficient are presented graphically and discussed.

AMS Subject Classification: 76M22
Key Words: inclined stretching surface, MHD, inclined magnetic field, unsteady, heat source

Received: March 3, 2021

[§]Correspondence author

1. Introduction

The study of the phenomenon of hydro-magnetic flow of an electrically conducting fluid over a continuous stretching surface has received the attention of many researchers due to its application in different fields of industrial manufacturing and engineering. The boundary layer flow on a continuous moving surface in a viscous medium was pioneered by Sakiadis [1]. Crane [2] was the first person to investigate boundary layer flow over a continuous solid surface moving with a constant speed.

Electrically conducting fluid in the presence of a magnetic field controls the rate of cooling and consequently affects the desired properties of the end product (Dessie and Kishan [3]), Choudhary et al. [4]. Rapid stretching for example, causes sudden solidification which destroys the characteristics of the final product Choudhary et al. [4], hence the need to control the cooling rate to get the desired product. The analysis of the stretching problem with constant surface temperature was carried out by Gupta and Gupta [5] as well as Vlegga [6]. Elshdawody and Aldawody [7] studied heat transfer of unsteady stretching surface with variable heat flux in the presence of heat source or sink. A vertical uniformly stretched permeable surface with heat generation, absorption and chemical reaction was studied by Chamka [8]. This study considered analytical solutions. A mixed convection fluid flow over an unsteady stretching surface in a porous medium was investigated by Imran et al. [9]. In this study the shooting method together with the sixth-order Runge-Kutta method was used.

In the present work we investigate inclined magnetic field on unsteady MHD and heat transfer over an inclined stretching surface in a porous medium in the presence of heat sources. Raju et al. [10] studied the effects of radiation, inclined magnetic field and cross diffusion on the flow over a stretching surface. The effects of inclined magnetic field with dissipation in non-Darcy medium was studied by Salawa and Dada [11]. Sandeep and Sugunamma [12] examined the inclined magnetic field and radiation effects of unsteady MHD free convection flow past a moving vertical plate in a porous medium. Hoyat et al. [13] studied the effects of inclined magnetic field with variable thermal conductivity. Further studies of the effects of inclined magnetic fields were considered by Dar and Elangovan [14] and Ibrahim et al. [15].

The study of flow over an inclined plate was studied by a considerable number of researchers which include among others Alam et al. [16], Alam et al. [17], Uddin [18], Buzuzi et al. [19], Buzuzi and Buzuzi [20], Buzuzi et al. [21]. Specifically the study of inclined stretching surfaces was also studied by a number of authors. Sterobel and Chen [22] investigated buoyancy effects on heat and mass transfer in boundary layer adjacent to inclined continuous sheets. Also Moutsoglou and Chen [23] studied buoyancy effects in boundary layer on inclined, continuous, moving sheet. Ali et al. [24] investigated the similarity solution of heat and mass transfer of flow over an inclined stretching sheet with dissipation and constant heat flux in the presence of magnetic field. Maruf et al. [25] analyzed the MHD free convection flow past an inclined stretching sheet in the presence of viscous dissipation and radiation.

The problem of inclined magnetic field on unsteady MHD and heat transfer over an inclined stretching surface in a porous medium in the presence of heat sources has received little attention. In this study, a spectral based method is used, this method has not been used in many studies, most of the studies considered analytical and finite-difference based methods. The successive linearization method is being used as an alternative to solving systems of equations arising in fluid mechanics, hence motivation for the current study.

The rest of the paper is organized as follows. In Section 2 we present the model formulation. Section 3 provides the method of solution. In Section 4 the numerical results are presented and discussed. Finally, the conclusion is given in Section 5.

2. Problem formulation

A two-dimensional unsteady flow of an incompressible fluid over a continuous moving inclined stretching surface subjected to an inclined magnetic field in a porous medium in the presence of heat source is considered. We assume that the surface is stretching with velocity $U_w = k_1 x/(1-ct)$ along the x-axis. Let the x-axis be directed along the continuous surface in the direction of motion and the y-axis be normal to the direction of motion. The stretching surface is inclined at an angle α to the horizontal and the magnetic field of uniform strength B_o is applied at an angle ϑ to the flow direction. Therefore the MHD unsteady convective boundary layer equations of continuity, momentum, energy take the following form:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\begin{aligned}\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma {B_o}^2 \sin^2\vartheta}{\rho}u - \frac{\nu}{K}u \\ + g\beta(T - T_\infty)\sin\alpha,\end{aligned} \tag{2}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{C_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{Q_0(T - T_\infty)}{\rho C_p}, \tag{3}$$

where u and v are the components of dimensional velocities along the x and y directions respectively, ρ is the fluid density, and ν is the coefficient of kinematic viscosity, β the coefficient of thermal expansion of fluid, K the permeability of the porous medium, σ is the electrical conductivity of the fluid, κ is the thermal diffusivity, C_p is the specific heat at constant pressure, B_o is the magnetic induction, T is the temperature of the fluid in the boundary layer, $Q_0 < 0$ is the heat absorption coefficient, $Q_0 > 0$ is the heat generation coefficient and g is gravitational acceleration.

The corresponding boundary conditions for the velocity, temperature fields are

$$\begin{aligned} &u = U_w(x,t),\ v = V_w(x,t), T = T_w(x,t),\ \text{at}\ \ y = 0,\\ &u \to 0,\ T = T_w(x,t),\ \text{as}\ \ y \to \infty, \end{aligned} \tag{4}$$

where U_w and V_w are the surface stretching velocity and transpiration velocity through the permeable surface respectively.

We choose a stream function $\psi(x,y)$ such that $u = \partial\psi/\partial y$, $v = -\partial\psi/\partial x$ which satisfies the continuity equation (1). In order to simplify the mathematical analysis of the problem we introduce the following dimensionless similarity variables [4]:

$$\begin{aligned} &\psi(x,y,t) = \sqrt{\nu x U_w} f(\eta),\\ &\eta = \sqrt{\frac{U_w}{\nu x}}y, T = T_\infty + \left(\frac{k_2}{k_1}\right) U_w \theta(\eta), \end{aligned} \tag{5}$$

where $f(\eta)$ is the dimensionless stream function, η is the similarity variable, $\theta(\eta)$ is the dimensionless temperature and y is the coordinate measured along the normal to the stretching surface. Using equations (5), equations (2) and (3) can be written in dimensionless form as

$$f''' + ff'' - \chi\left(\frac{\eta}{2}f'' + f'\right) - f'^2 + \tau_1\theta\sin\alpha - \xi f' - M\sin^2\vartheta f' = 0, \tag{6}$$

$$\frac{1}{P_r}\theta'' + f\theta' - \theta f' - \chi\left(\theta + \frac{\eta}{2}\theta'\right) + Ec(f'')^2 + \lambda_1\theta = 0. \tag{7}$$

The accompanying boundary conditions now become

$$f(0)=0,\ f'(0)=1,\quad \theta(0)=1 \ \text{ on } \ \eta=0,$$
$$f'(\infty)=0,\quad \theta(\infty)=0 \ \text{ as } \ \eta\to\infty, \tag{8}$$

where the prime ($'$) denotes differentiation with respect to η, $\gamma = k_1/(1-ct)$, $\chi = c/k_1$ is the unsteadiness parameter, $\tau_1 = G/Re_x^2$ is the bouyancy parameter, $\lambda_1 = \delta/\gamma$ is the dimensionless heat source/sink parameter, $\delta = Q_0/\rho C_p$, $P_r = (\mu C_p)/\kappa$ is the Prandtl number, $M = (\sigma B_0^2 \nu Re_x)/(\rho U_w^2)$ is the magnetic parameter, $\xi = 1/(Da.Re_x)$ is the permeability parameter, $E_c = U_w^2/(C_p(T_w - T_\infty))$ is the Eckert number and G, Q_0, Da are the local Grashof number, heat absorption coefficient and the local darcy number respectively.

It is of interest to engineers to consider the two flow characteristics, namely, the skin friction C_f, and local Nusselt number Nu_x defined as

$$C_f = \frac{2\mu \frac{\partial u}{\partial y}|_{y=0}}{\rho u_w^2} = \frac{2f''(0)}{\sqrt{Re_x}},$$
$$Nu_x = \frac{-x\frac{\partial T}{\partial y}|_{y=0}}{T_w - T_\infty} = -\theta'(0)\sqrt{Re_x},$$

where Re is the Reynolds number.

3. Method of solution

Equations (6)-(7) are solved using the successive linearization method (see [26]). The velocity and temperature variables f and θ are expressed in the form

$$f(\eta) = f_i(\eta) + \Sigma_{N=0}^{i-1} f_N(\eta), \tag{9}$$
$$\theta(\eta) = \theta_i(\eta) + \Sigma_{N=0}^{i-1} \theta_N(\eta), \tag{10}$$

where f_i and θ_i $(i = 1, 2, 3, \ldots)$ are such that $\lim_{i\to\infty} f_i(\eta) = \lim_{i\to\infty} \theta_i(\eta) = 0$ and f_N and θ_N are obtained by recursively solving the linear part of the equations obtained by substituting (9-10) into (6) and (7). The non-linear part of f_i and θ_i are considered small and thus are neglected. The initial guesses $f_0(\eta)$ and $\theta_0(\eta)$ are taken to be

$$f_0(\eta) = 1 - \exp(-\eta), \quad \theta_0(\eta) = \exp(-\eta). \tag{11}$$

The initial approximation are so chosen to satisfy the boundary conditions (8). The subsequent solutions for f_i and θ_i $(i \geq 1)$ are obtained by successively solving the linearized form of the equations. We seek the solution to the following linearized equations

$$f_i''' + a_{1,i-1}f_i'' + a_{2,i-1}f_i' + a_{3,i-1}f_i + a_{4,i-1}\theta_i = r_{1,i-1}, \tag{12}$$

$$\begin{aligned} b_{1,i-1}\theta_i'' + b_{2,i-1}\theta_i' + b_{3,i-1}\theta_i + b_{4,i-1}f_i'' + b_{5,i-1}f_i' \\ + b_{6,i-1}f_i = r_{2,i-1}, \end{aligned} \tag{13}$$

subject to the corresponding boundary conditions

$$\begin{aligned} &f_i(0) = f_i'(0) = f_i'(\infty) = 0, \\ &\theta(0) = \theta(\infty) = 0, \end{aligned} \tag{14}$$

where

$$\begin{aligned}
&a_{1,i-1} = \Sigma_{N=0}^{i-1} f_N - \frac{\chi\eta}{2}, \quad a_{2,i-1} = -\left(\xi + \chi + M\sin^2\vartheta\right), \\
&a_{3,i-1} = \Sigma_{N=0}^{i-1} f_N'' - 2\Sigma_{N=0}^{i-1} f_N, \quad a_{4,i-1} = \tau\sin\alpha, \\
&b_{1,i-1} = \frac{1}{Pr}, \quad b_{2,i-1} = \Sigma_{N=0}^{i-1} f_N - \frac{\chi\eta}{2}, \\
&b_{3,i-1} = \lambda_1 - \Sigma_{N=0}^{i-1} f_N' - \chi, \\
&b_{4,i-1} = 2Ec\Sigma_{N=0}^{i-1} f_N'', \quad b_{5,i-1} = -\Sigma_{N=0}^{i-1}\theta_N, \quad b_{6,i-1} = \Sigma_{N=0}^{i-1}\theta_N', \\
&r_{1,i-1} = \left(\Sigma_{N=0}^{i-1}\right)^2 + \xi\Sigma_{N=0}^{i-1} f_N + \xi\left(\frac{\eta}{2}\Sigma_{N=0}^{i-1} f_N'' + \Sigma_{N=0}^{i-1} f_N'\right) \\
&\quad + M\sin^2\vartheta\Sigma_{N=0}^{i-1} f_N' - \Sigma_{N=0}^{i-1} f_N''' - \Sigma_{N=0}^{i-1} f_N \Sigma_{N=0}^{i-1} f_n' \\
&\quad - \tau\sin\alpha\Sigma_{N=0}^{i-1}\theta_N, \\
&r_{2,i-1} = \Sigma_{N=0}^{i-1}\theta_N\Sigma_{N=0}^{i-1} f_N' + \chi\left(\Sigma_{N=0}^{i-1}\theta_N + \frac{\eta}{2}\Sigma_{N=0}^{i-1}\theta_N'\right) \\
&\quad - \frac{1}{Pr}\Sigma_{N=0}^{i-1}\theta_N'' - \Sigma_{N=0}^{i-1} f_N\Sigma_{N=0}^{i-1}\theta_N' - Ec\left(\Sigma_{N=0}^{i-1} f_N''\right)^2 \\
&\quad - \lambda_1\Sigma_{N=0}^{i-1}\theta_N.
\end{aligned}$$

The solution for f_i and θ_i $(i \geq 1)$ are obtained iteratively by solving equations (12)-(13). The approximate solution for $f(\eta)$ and $\theta(\eta)$ are obtained as

$$f(\eta) \approx \Sigma_{m=0}^{M} f_m(\eta), \tag{15}$$

$$\theta(\eta) \approx \Sigma_{N=0}^{M} \theta_m(\eta), \tag{16}$$

where M is the order of SLM approximation. Equations (12)-(13) were solved using the Chebyshev spectral collocation method. For more detailed analysis of SLM (see [26]).

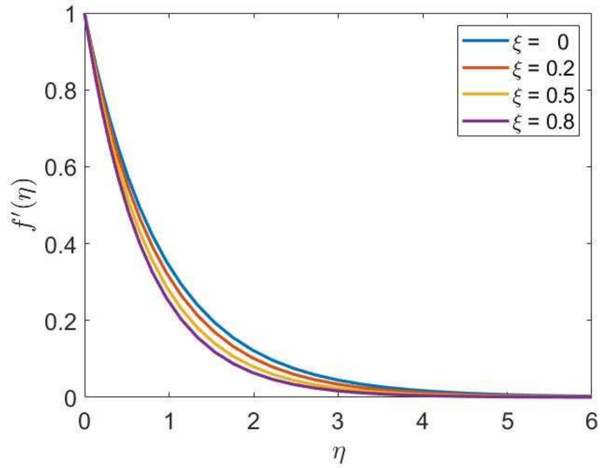

Figure 1: Velocity profile for different values of ξ

4. Results and discussion

The problem of an inclined stretching surface in a porous medium with heat source and variable magnetic field angle is studied. In order to analyze the physical interpretation of the problem, computations have been carried out for the various flow parameters associated with the flow such as the angle of the magnetic field inclination (ϑ), the angle of channel inclination (α), the unsteadiness parameter (χ), the permeability parameter (ξ), the Prandtl number (Pr), magnetic field parameter M and heat source/sink parameter (λ_1). The numerical results are obtained and analyzed graphically as depicted in Figures 1-15.

Figure 1 shows the effect of varying the permeability parameter on velocity profiles, increasing the permeability parameter has an effect of reducing the velocity profiles. It is assumed that the value $\xi = 0$ refers to a non-porous media, and $\xi > 0$ indicates porosity. The velocity profiles reduce with increasing ξ because the number of pores reduce thereby reducing fluid movement. The profiles are less enhanced due to the presence of the inclined plane which tend to assist the fluid flow.

In Figure 2 shows the effect of changing the magnetic inclination parameter. Increasing ϑ result in the decrease in velocity profiles. When the angle of magnetic inclination is zero, it this tends to assist the flow and it will be parallel

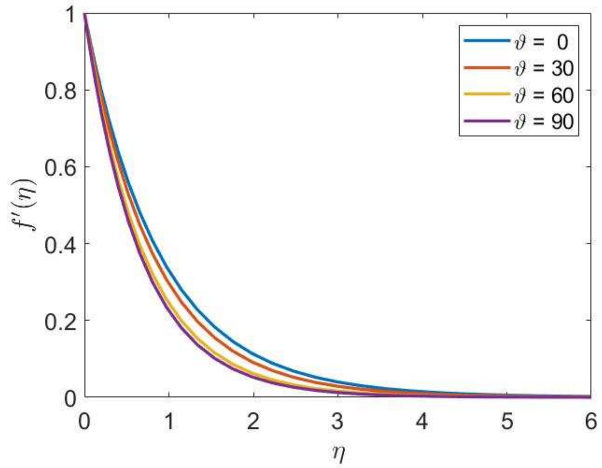

Figure 2: Velocity profile for different values of ϑ.

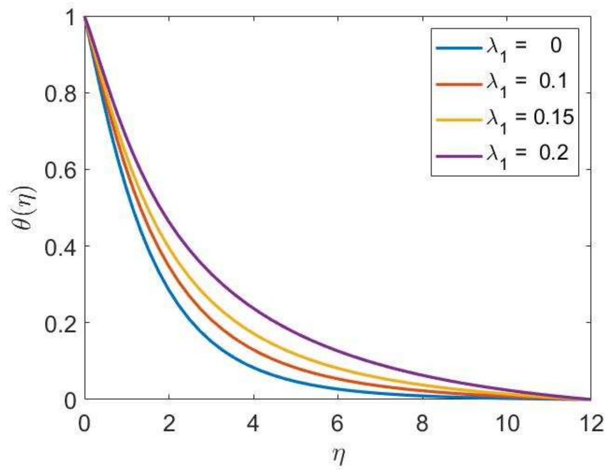

Figure 3: Temperature profile for different values of λ_1

and in the same direction as the fluid flow. When the angle increases, this has an effect of retarding the fluid flow. When $\vartheta = 90^o$, the magnetic field will be perpendicular to the direction of the fluid flow causing the fluid velocity to be small.

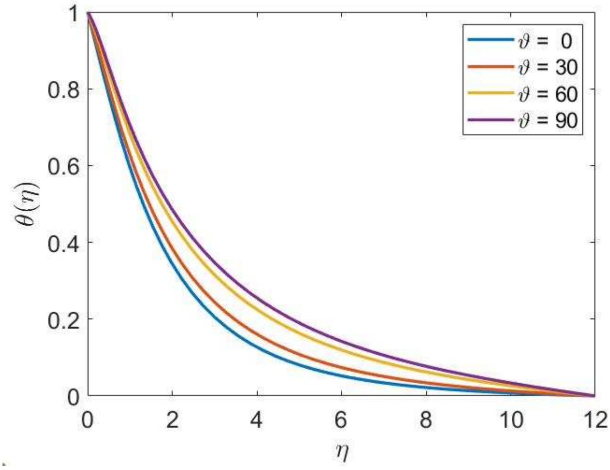

Figure 4: Temperature profile for different values of ϑ

In Figure 3, it shown that increasing λ_1 has a result of increasing the temperature profile across the boundary layer. This is consistent with previous results in the literature.

Figure 4 shows that effect of varying the magnetic field inclination. In this case the angle of channel inclination is large, the increase in ϑ tend to assist the flow resulting in the increase in velocity causing fluid temperature to increase.

Figure 5 shows the effect of varying the permeability parameter ξ on temperature profiles. Increasing ξ has an effect of increasing temperature profiles, this is caused by the interaction of fluid particles and pores that are randomly situated in the flow regime. This interaction cause fluid particles to follow a "chaotic path" causing viscous dissipation to occur.

Figure 6 depicts the effect of varying the permeability parameter on the skin friction coefficient $f''(0)$. Increasing ξ result in the decrease of skin friction coefficient, as the fluid moves down the inclined plane the velocity is retarded by the porous media causing skin friction to decrease. It is also noted that skin friction coefficient decrease with increasing magnetic inclination parameter ϑ. As ϑ increases, it has an effect of decreasing the fluid velocity thereby decreasing the skin friction coefficient at the surface.

Figures 7-12 show the effect of varying parameters ϑ, Pr, M, Ec, χ, α on the convergence and accuracy of the successive linearization method (SLM). The successive linearization method has been shown to be accurate and converges

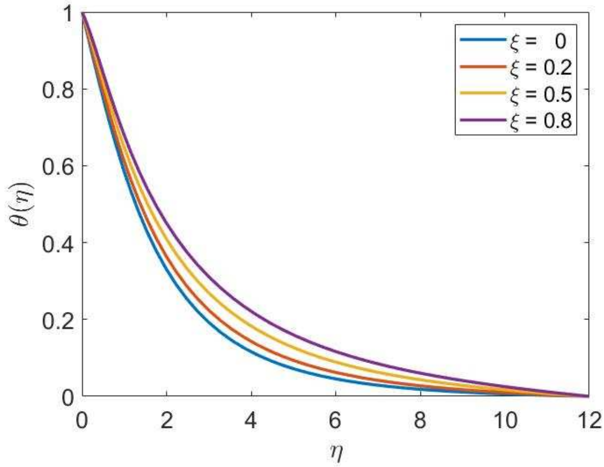

Figure 5: Temperature profile for different values of ξ

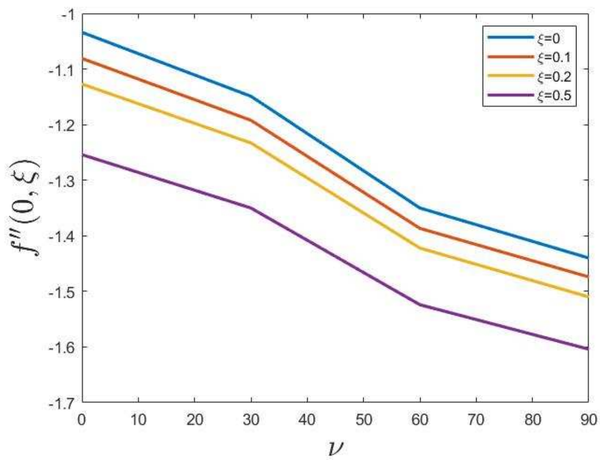

Figure 6: Effect of varying ξ and ϑ on the skin friction

faster than many traditional finite-difference methods. This method has been widely used in solving ordinary differential equations arising from fluid flow. In this study we show how varying the parameters affect the accuracy of the SLM. In Figure 7 the increase in ϑ tend to stabilize the convergence, the residual

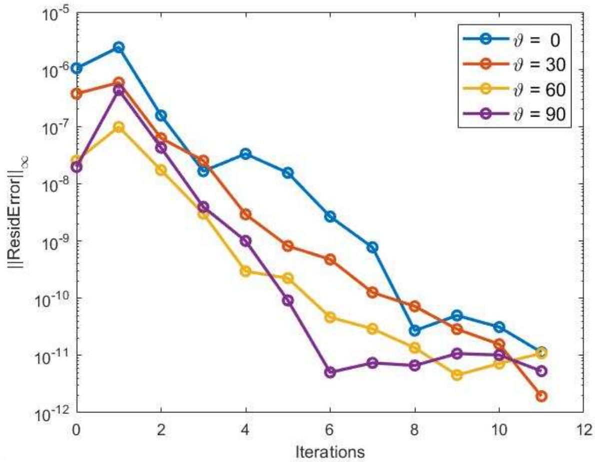

Figure 7: Convergence of SLM with varying ϑ

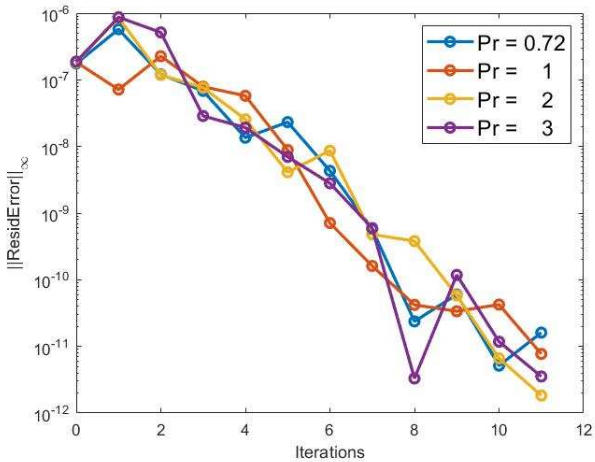

Figure 8: Convergence of SLM with varying Pr

error going as low as 10^{-12}. In Figure 8, increasing the Prandtl number delays the convergence up to eight iterations. In Figure 9, increasing the M has little effect in the convergence as it is noticed just after four iterations. Also residual errors are as low as 10^{-12}. In Figures 10-12 shows the increase in Ec, χ, α result

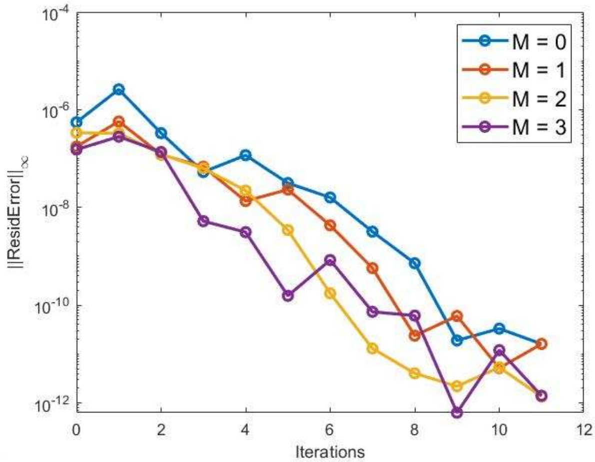

Figure 9: Convergence of SLM with varying M

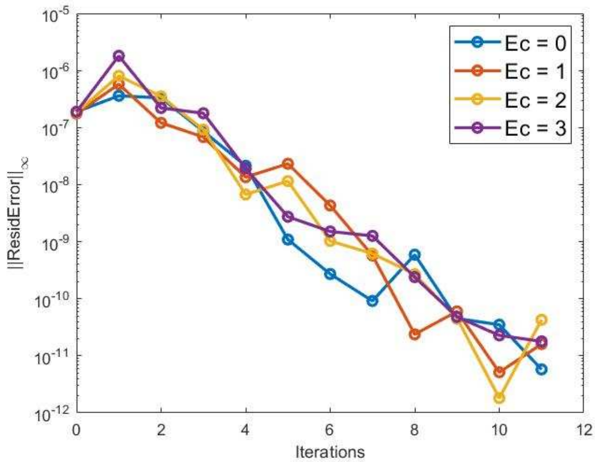

Figure 10: Convergence of SLM with varying Ec

in stabilising the SLM, the residual errors gradually decrease as expected. The increase in each of the parameters show a slight delay in the convergence of the method. In general the method does not deteriorate with increasing iterations. This method can be used as an alternative method in the solving of ordinary

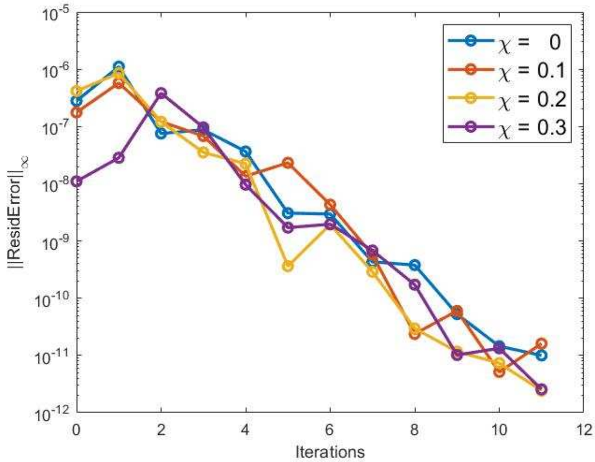

Figure 11: Convergence of SLM with varying χ

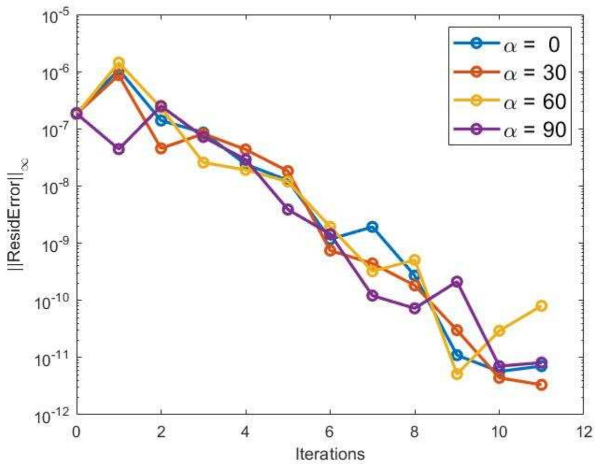

Figure 12: Convergence of SLM with varying α

differential equations.

In Figure 13, it is noticed that increasing the unsteadiness parameter χ result in the decrease in the skin friction coefficient. When χ increase, the fluid velocity decrease thereby decreasing the skin-friction coefficient. It is also noted

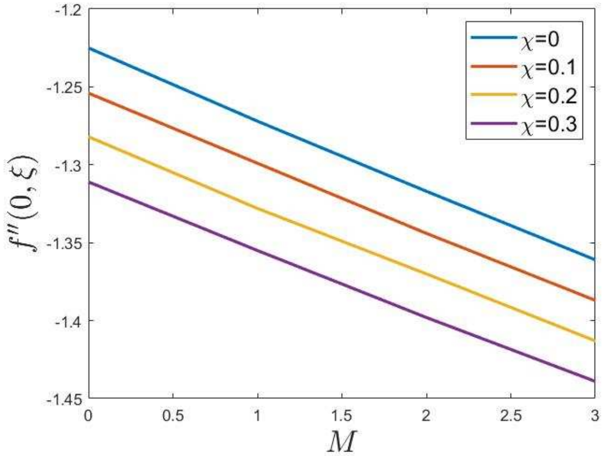

Figure 13: Effect of χ and M on the skin friction

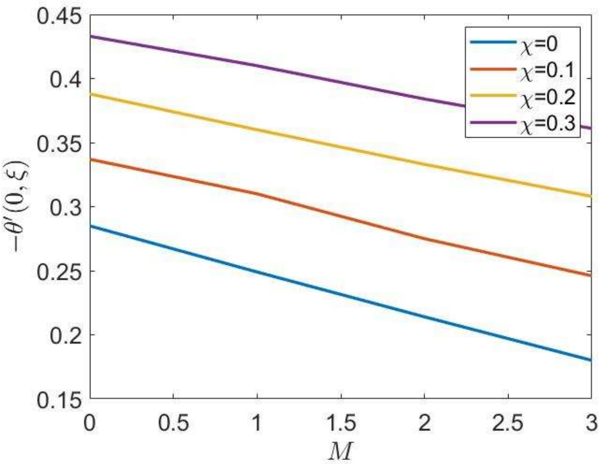

Figure 14: Effect of χ and M on the heat flow coefficient

that increasing the magnetic parameter M result in the decrease in the in the skin friction coefficient. The magnetic field strength reduce the fluid velocity thereby decreasing the skin friction coefficient.

Figure 14 shows the effect of varying the unsteadiness parameter χ on the

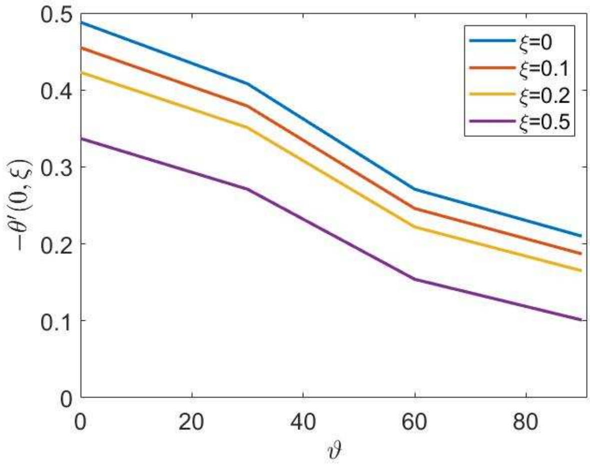

Figure 15: Effect of ϑ and ξ on the heat transfer coefficient

heat transfer coefficient. Increasing the unsteadiness parameter result in the increase of the heat transfer coefficient. Increasing χ result in the decrease in fluid velocity, this then decreases heat transfer at the surface. The increase in the magnetic parameter M has the same effect of decreasing the fluid velocity thereby decreasing the heat transfer coefficient.

Figure 15 shows the effect of varying the permeability parameter ξ on the heat transfer coefficient. Increasing ξ result in the decrease in heat transfer coefficient, increasing ξ causes the fluid velocity to decrease enhancing heat transfer. Increasing the magnetic inclination parameter result in the increase in the heat transfer parameter. Increase ϑ has the same effect of decreasing the fluid velocity thereby decreasing the heat transfer coefficient.

5. Concluding remarks

The present paper deals with the numerical analysis on unsteady MHD flows and heat transfer over an inclined stretching channel in a porous medium with heat sources in the presence of inclined magnetic field. The following conclusions emerged from the present numerical study.

(i) Velocity decreases with increasing permeability parameter ξ and inclination angle of the magnetic field ϑ.

(ii) The temperature increases with increasing heat source parameter λ_1, inclination angle of the magnetic field ϑ and the permeability parameter ξ.

(iii) Increasing the permeability parameter ξ, inclination angle of the magnetic field ϑ, unsteadiness parameter χ and the magnetic field parameter M have the effect of decreasing the skin friction.

(iv) Increasing the unsteadiness parameter χ enhances the heat transfer coefficient.

(v) Increasing the magnetic parameter M, the permeability parameter ξ and the inclination angle of the magnetic field ϑ decreases the heat transfer coefficient.

(vi) The inclination angle of the magnetic field ϑ, Eckert number Ec, unsteadiness parameter χ and the channel inclination angle α have a stabilizing effect on the successive linearization method (SLM) which was applied in the current study.

References

[1] B. C. Sakiadis, Boundary layer behaviour on continuous solid surfaces: Boundary layer equations for two dimensional and axisymmetric flow, *AIChE J.*, **7**, No 1 (1961), 26-28.

[2] L. J. Crane, Flow past a stretching plane, *Z. Angew. Math. Phys.*, **21** (1970), 645-647.

[3] H. Dessie, N. Kishan, MHD effects onheat transfer over stretching sheet embedded in porous medium with variable viscosity, viscous dissipation and heat source/sink, *Ain Shams Eng. J.*, **5** (2014), 967-977.

[4] M. K. Choudhary, S. Chaudhary, R. Sharma, Unsteady MHD flow and heat transfer over a stretching permeable surface with suction or injection, *Procedia Eng.*, **127** (2015), 703-710.

[5] P. S. Gupta, A. S. Gupta, Heat and mass transfer on a stretching sheet with suction or blowing, *Canadian J. Chem. Eng.*, **55**, No 6 (1977), 744-746.

[6] J. Vleggaar, Laminar boundary layer behaviour on continuous solid surfaces, *Chem. Eng. Sci.*, **32** (1977), 1517-1525.

[7] E. M. A. Elbashbeshy, D.A. Aldawody, Heat transfer over an unsteady stretching surface with variable heat flux in the presence of a heat source or sink, *Comput. Math. with Appl.*, **60** (2010), 2806-2811.

[8] A. J. Chamkha, MHD flow of a uniformly stretched vertical permeable surface in the presence of heat generation/absorption and a chemical reaction, *Int. Commun. Heat Mass Transf.*, **30** (2003), 413-422.

[9] S. M. Imran, S. Asghar, M. Mushtaq, Mixed convection flow over an unsteady stretching surface in a porous medium with heat source, *Math. Probl. Eng.*, **2012** (2012), 1-16; DOI: 10.1155/2012/485418.

[10] C. S. K. Raju, N. Sandeep, C. Sulochana, V. Suganamma, M. Jayachandra, Radiation, inclined magnetic field and cross-diffusion effects on flow over a stretching surface, *J. Nigerian Math Soc.*, **34** (2015), 169-180.

[11] S. O. Salawu, M. S. Dada, Radiation heat transfer of variable viscosity and thermal conductivity effects on inclined magnetic field with dissipation in a non-Darcy medium, *J. Nigerian Math Soc.*, **35** (2016), 93-106.

[12] N. Sandeep and V. Sugunamma, Radiation, inclined magnetic field effects on unsteady hydromagnetic free convection flow past an impulsively moving vertical plate in a porous medium, *J. Appl. Fluid. Mech.*, **7**, No 2 (2014), 275-286.

[13] T. Hoyat, A. Shafiq, A. Alsaedi and Asghar, Effect of inclined magnetic field in flow of third grade fluid with variable thermal conductivity, *AIP Adv.*, **5** (2015), 1-13; DOI: 10.1063/1.4928321.

[14] A. A. Dar, K. Elangovan, Thermal diffusion, radiation and inclined magnetic field effects on oscillatory flow in an asymmetric channel in presence of heat source and chemical reaction, *J. Nigerian Math Soc.*, **35** (2016), 488-509.

[15] F. S. Ibrahim, A. M. Elaiw, A. A. Bakr, Effect of the chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi infinite vertical permeable moving plate with heat source and suction, *Commun. in Nonlin. Sci. and Numer. Simul.*, **13** (2008), 1056-1066.

[16] M. S. Alam, M. M. Rahman, A. Satter, Effects of variable suction and thermophoresis on steady MHD combined free-forced convective heat and mass transfer flow over a semi-infinite permeable inclined plate in the presence of thermal radiation, *Int. J. Therm. Sci.*, **47**, No 6 (2008), 758-765.

[17] M. S. Alam, M. M. Rahman, A. Satter, Transient magnetohydrodynamic free convective heat and mass transfer flow with thermophoresis past a radiate inclined permeable plate in the presence of variable chemical reaction and temperature dependent viscosity, *Nonlinear Anal.: Model. Control*, **14** (2009), 3-20.

[18] M. J. Uddin, Convective flow of micropolar fluids along an inclined flat plate with variable electric conductivity and uniform surface heat flux, *Daffodil Int. Univ. J. of Sci. and Techn.*, **6**, No 1 (2011), 69-79.

[19] G. Buzuzi, J. B. Munyakazi, K. C. Patidar, A fitted numerical method to investigate the effect of various parameters on an MHD flow over an inclined plate, *Numer. Methods Partial Differ. Equ.* (2015), 107-120; DOI: 10.1002/num.21986.

[20] G. Buzuzi, A. N. Buzuzi, Unsteady MHD convection flow and heat transfer past a vertical inclined plate in a porous medium with variable plate temperature with suction in a slip flow regime, *Int. J. Appl. Math.*, **32**, No 2 (2019), 205-218; DOI: 10.12732/ijam.v32i2.4.

[21] G. Buzuzi, A. N. Buzuzi, T. Nyamayaro, W. Manamela, K. S. Ramolotja, The study of magneto hydrodynamic free convective flow and heat transfer in a porous medium with heat generation, radiation absorption and chemical reaction, *Int. J. Appl. Math.*, **33**, No 4 (2020), 733-756; DOI: 10.12732/ijam.v33i4.15.

[22] F. A. Strobel, T. S. Chen, Bouyancy effects on heat and mass transfer in booundary layers adjacent to inclined continuous moving sheet, *Numer. Heat Transfer A*, **3**, No 4 (1980), 461-481.

[23] A. Moutsoglou, T. S. Chen, Bouyancy effects in boundary layer on inclined continuous moving sheets, *J. Heat Transfer*, **102**, No 2 (1980), 371-373.

[24] M. Ali, M. A. Alim, M. S. Alam, Similarity solution of heat and mass transfer fow over an inclined stretching sheet with viscous dissipation and constant heat flux in presence of magnetic field, *Procedia Eng.*, **105** (2015), 557-569.

[25] M. Hassan, E. Karim, A. Samad, MHD free convection flow past an inclined stretching sheet with considering viscous dissipation and radiation, *Open J. Fluid Dyn.*, **7** (2017), 152-168.

[26] G. Makanda, D. Makinde, P. Sibanda, Natural convection of viscoelastic fluid from a cone-embedded in a porous mwdium with viscous dissipation, *Math. Probl. Eng.*, **2013**, (2013), 1-11; DOI: 10.1155/2013/934712.