

# Flexural Behaviour of Continuous Steel Beam Prestressed Via Externally Unbonded Tendons: Numerical Modelling, Parametric Study and Statistical Analysis



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**Abstract** Prestressed steel beam has the advantages of lightweight sections; this makes them economical and feasible solution in various practical situations. Prestressing the steel beams using externally unbonded tendons improves their load-carrying capacity and serviceability performance. This paper reports the flexural behaviour of continuous steel beams strengthened by prestressed tendons under concentrated load using finite element software, ANSYS. Mid-span deflection of the beam is essential to decide its functionality requirement. The effect of tendon eccentricity and prestressing force on the mid-span deflection has been analysed in detail through parametric studies. Furthermore, a simplified model derived from multivariable regression analysis is proposed for predicting the deflection of the prestressed continuous steel beam. The proposed model is based on applied load, the eccentricity of tendons and prestressing force with a high coefficient of determination ( $R^2 > 0.99$ ). Numerical results show 15–27% improvement in flexural capacity in terms of reduction in mid-span deflection.

**Keywords** Continuous steel beam · Finite element modelling · Parametric study · Statistical analysis

## 1 Introduction

Metal bridges are dated back to 1779, but the first steel bridge was constructed in 1823, a cable suspension bridge [1]. Over time, elements of steel bridges get dilapidated, resulting in structural failure, causing human, economic, environmental and

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social losses [2]. Therefore, careful inspection, evaluation, repairs and rehabilitation of older steel bridges are an important and specific task for preserving the cultural heritage and reducing the losses [3, 4]. The post-tensioning method is widely adopted for strengthening and rehabilitating existing concrete bridge girders [5, 6]. A similar approach can also apply to steel bridges. The prestressed steel truss-supported hanger at Melsbroek Airport in Belgium and Livery street bridge in England are some of the live examples. For new steel girder bridges, this technique leads to 15–20% material saving, which reduces the overall cost and provides the solution of sustainable structures by saving the material [7–9].

In the literature, prestressed steel beam is studied by many researchers. Bradford [10] presents the design chart based on an analysis method for the elastic buckling of prestressed steel girders. Vincenzo [11] shows the superiority of prestressed steel structure in resistance, inexpensiveness and execution simplicity. Park et al. [12] studied the flexural behaviour of prestressed simply supported and continuous steel beams and reported that the inclusion of prestressing in steel I-beam increases its ultimate load-carrying capacity. If provided with section enhancement at inner supports, an additional 30–35% increment was noted. Belletti and Gasperi [13] analyse the behaviour of the medium span (35–45 m) I-shaped steel beam prestressed with external tendons design parameter: the number of deviators and amount of prestressing force. They found that beams with bracing at the top and bottom have better performance with respect to a beam having bracing only at flanges. Ponnada and Vipparthy [14, 15] pointed out that prestressing techniques are more feasible for unsymmetrical I sections than the symmetrical section. In another work, the author presented the method for estimating the deflection of the prestressed steel beam. Zhang [16] present the analytical solution of symmetric and antisymmetric lateral-torsional buckling of prestressed steel girder. The prestressing tendons were placed inside the depth of the beam itself. The author concluded that critical moments under equal end moments could be determined by symmetric buckling. Also, the critical prestressing force can be obtained by symmetric or antisymmetric buckling, depending upon the relation between eccentricity and its threshold value. Ren et al. [17] present a method and parametrically study the effect of prestressing force and eccentricity of tendons on the flexural behaviour of deep single-span I-beam. The author concluded that the tensile rigidity ratio between sub-tendons made no changes to the increase in sub-tendons strains. Hadjipantelis et al. [18] reported that prestressing technique enhances the moment-carrying capacity of the cold-formed steel beams.

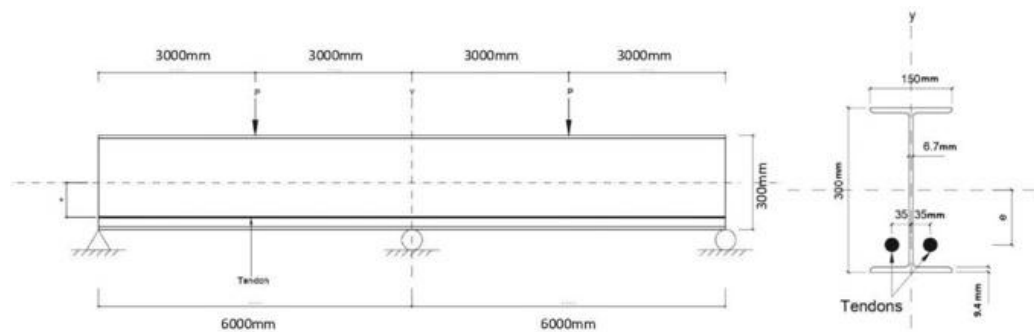
Most of the earlier study focuses on the analysis of simply supported prestressed single-span I-beam. Not much literature is available on the behaviour of a continuous prestressed steel beam, having tendons housed within the depth of the beam. For the rational design of continuous prestressed steel beams, a good understanding of the behaviour of continuous steel beams with external prestressing is needed. The present study addressed this research gap and investigated the flexural behaviour of continuous prestressed steel beams with varying tendon eccentricity (0–100 mm) and prestressing force (0–50% of yield stress). The finite element model of the beams

has been generated and analysed. In addition, a simplified mathematical equation is proposed for predicting the mid-span deflection in analytical studies.

## 2 Numerical Investigation

### 2.1 Analysed Prestressed Steel Beam

The geometrical dimensions of the beam and tendons are illustrated in Fig. 1. The beam is a two-span continuous beam with an equal span of 6 m each. The cross-section is ISMB300 symmetrical about  $Y$ -axis conforming to IS 808:1089 [19], and grade structural steel is E450 as per the IS 2062:2011 [20] as shown in Fig. 1. Each steel  $I$ -beam is provided with the two prestressing tendons of diameter 25 mm, located at a constant eccentricity of 35 mm about the  $Y$ -axis and varying eccentricity about the  $X$ -axis on both sides. The physical and mechanical properties of the material of beams and tendons are summarized in Table 1.



**Fig. 1** Schematic diagram of studied beams

**Table 1** Material property of the beam and tendons

| S No. | Properties                | Values                                |
|-------|---------------------------|---------------------------------------|
| 1     | Density                   | 7850 kg/m <sup>3</sup>                |
| 2     | Modulus of rigidity       | $0.769 \times 10^5$ N/mm <sup>2</sup> |
| 3     | Modulus of elasticity     | $2 \times 10^5$ N/mm <sup>2</sup>     |
| 4     | Poisons ratio             | 0.3                                   |
| 5     | Yield strength of beam    | 450 N/mm <sup>2</sup>                 |
| 6     | Yield strength of tendons | 1400 N/mm <sup>2</sup>                |
| 7     | Coefficient of expansion  | $12 \times 10^{-6}/^{\circ}\text{C}$  |

## 2.2 *Finite Element Modelling*

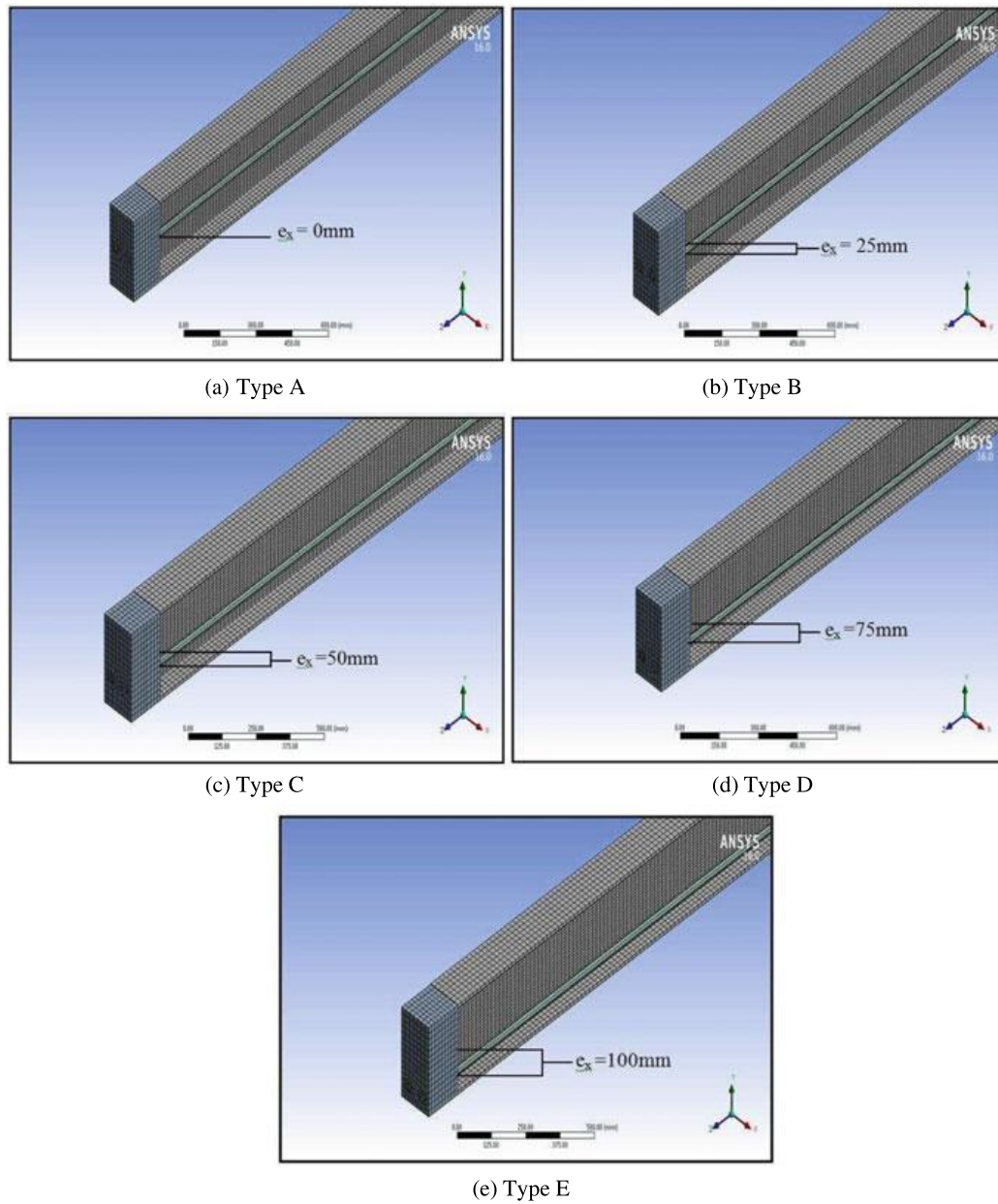
Finite element analysis was carried out using the ANSYS software package. The beam has been modelled using the 4-node shell element Shell 181, whereas the Link180 element is employed for tendons. Shell 181 elements have six degrees of freedom, and hence, it will precisely capture the model beam's behaviour. The appropriate mesh size was obtained through mesh convergence studies, and a mesh size of  $10 \times 10$  mm was found pertinent. Throughout the study, the mesh density has been taken to be 10 mm.

The mechanical properties of the beam model have been defined using the elastic–plastic model with isotropic hardening behaviour. First of all, a prestressed simply supported beam is modelled in ANSYS, in which one end is a hinge, and the other end is roller support. The developed FE model of the simply supported beam is validated by the mid-span deflection results of Park et al. [12], i.e. 8.46 mm with an error percentage of 8.74%, hence acceptable. Afterwards, the developed model is modified to the continuous beam by incorporating roller support at the mid-span to make a two-span prestressed continuous beam and in housing the tendon within the web. The hinge support is modelled by restricting the translational motion about the  $Y$ - and  $Z$ -axes, whereas translational motion about  $Y$ -axis is restricted at the other end support for roller support.

Figure 2 shows the tendon eccentricity in different FE models of prestressed continuous steel beams. The prestressing forces are applied as initial stresses in the percentage of yield strength of the beam. In all the beams, tendons are placed at the constant distance from the vertical  $Y$ -axis of the beam cross-section, and their eccentricity with respect to  $X$ -axis varies from 0 to 100 mm in steps of 25 mm. The connection between the tendon and plate attached with the beam was modelled as bonded; i.e. slip loss and anchorage loss are not considered. The constant point loads are applied at the middle of each span from the end support, as shown in Fig. 1.

## 2.3 *Parametric Study*

For investigating the flexural behaviour, beams analysed were (a) beams with constant eccentricity and varying prestressing force and (b) beams with constant prestressing force and varying eccentricity. Beams are labelled based on the eccentricity of the tendon wire and prestressing force value. Beam type A is the beams with tendon eccentricity 0 mm. Similarly, types B, C, D and E beams have tendons eccentricity of 25, 50, 75 and 100 mm, respectively. The beams are prestressed with varying prestressing force from 0 to 50% of beam yield strength in steps of 12.5%, as shown in Table 2 and Fig. 3a–e.



**Fig. 2** Tendon location in different FE model of prestressed continuous steel beams

### 3 Results and Discussion

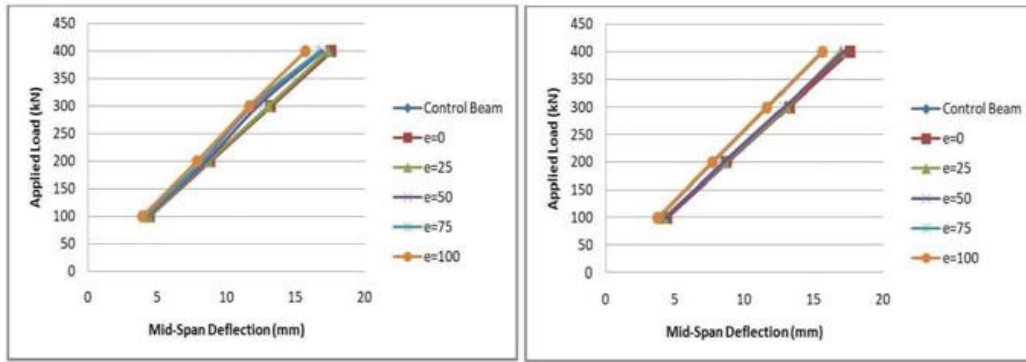
#### 3.1 Effect of Tendon Eccentricity

The variation of mid-span deflection with varying the tendons eccentricity and different constant prestressing forces is shown in Fig. 3a–e. It can be seen that at a particular prestressing force, as the eccentricity of the tendons about the  $x$ -axis increase, the mid-span deflection curve moves away from the control beam and

**Table 2** Different types of beams analysed

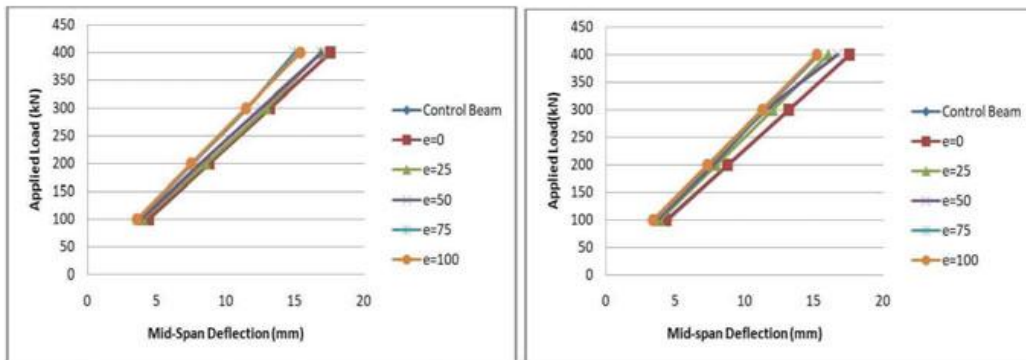
| Beam type            | Tendon eccentricity (mm) | Prestressing force (%) |
|----------------------|--------------------------|------------------------|
| A                    | –                        | –                      |
| B <sub>0,0</sub>     | 0                        | 0                      |
| B <sub>0,50</sub>    | 0                        | 12.5                   |
| B <sub>0,100</sub>   | 0                        | 25                     |
| B <sub>0,150</sub>   | 0                        | 37.5                   |
| B <sub>0,200</sub>   | 0                        | 50                     |
| C <sub>25,0</sub>    | 25                       | 0                      |
| C <sub>25,50</sub>   | 25                       | 12.5                   |
| C <sub>25,100</sub>  | 25                       | 25                     |
| C <sub>25,150</sub>  | 25                       | 37.5                   |
| C <sub>25,200</sub>  | 25                       | 50                     |
| D <sub>50,0</sub>    | 50                       | 0                      |
| D <sub>50,50</sub>   | 50                       | 12.5                   |
| D <sub>50,100</sub>  | 50                       | 25                     |
| D <sub>50,150</sub>  | 50                       | 37.5                   |
| D <sub>50,200</sub>  | 50                       | 50                     |
| E <sub>75,0</sub>    | 75                       | 0                      |
| E <sub>75,50</sub>   | 75                       | 12.5                   |
| E <sub>75,100</sub>  | 75                       | 25                     |
| E <sub>75,150</sub>  | 75                       | 37.5                   |
| E <sub>75,200</sub>  | 75                       | 50                     |
| F <sub>100,0</sub>   | 100                      | 0                      |
| F <sub>100,50</sub>  | 100                      | 12.5                   |
| F <sub>100,100</sub> | 100                      | 25                     |
| F <sub>100,150</sub> | 100                      | 37.5                   |
| F <sub>100,200</sub> | 100                      | 50                     |

tends towards lower values. This characteristic reduction in deflection is attributed to the stress reduction caused by the opposite moment developed by prestressed eccentric tendons. External point load generates the positive and negative moments at the mid-span and ends, respectively. The tendons oppose these generated positive moments at mid-span by their negative moment resulting in net reduction in stress below the neutral axis, leading to lower deflection. At the internal supports in continuous beam, negative moment generated by the prestressing force integrated with the negative moment due to external load and causes an increase in the moment values.



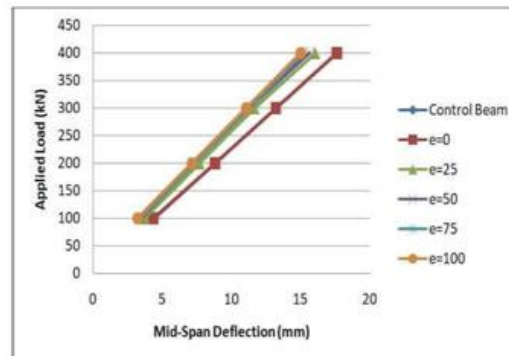
(a) PF=0%

(b) PF=12.5%



(c) PF=25%

(d) PF=37.5%

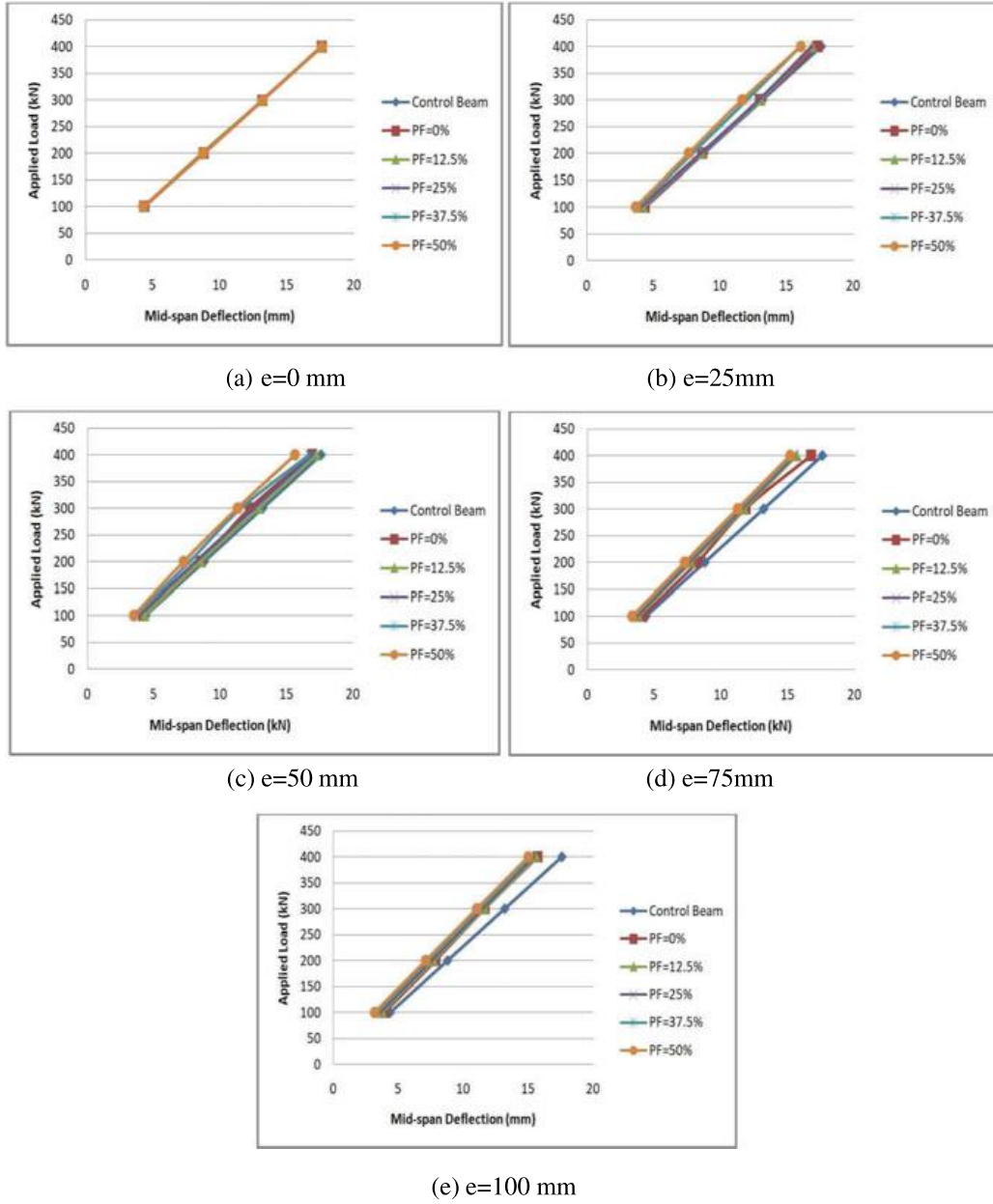


(e) PF=50%

**Fig. 3** Variation of mid-span deflection at different tendon eccentricity with constant prestressing force

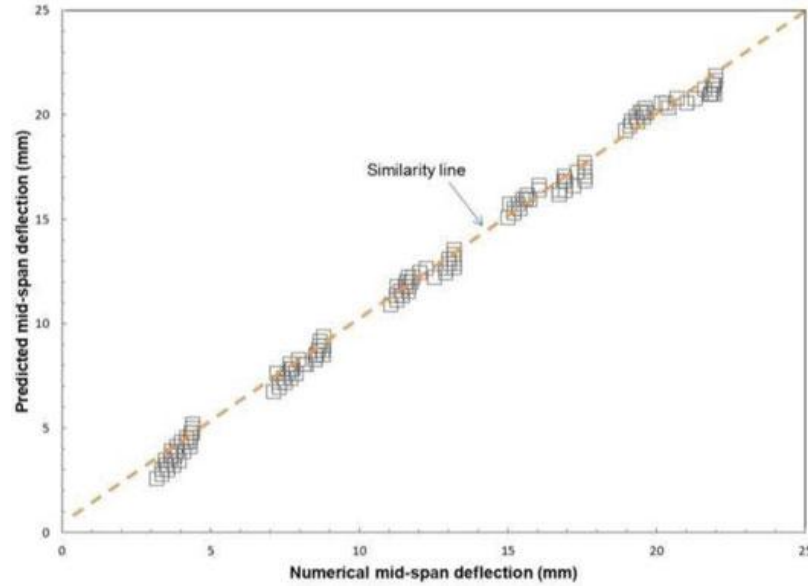
### 3.2 Effect of Prestressing Force

Figure 4a–e shows the variation of mid-span deflection in the continuous beam with varying prestressing force at different constant initial tendon eccentricities. In all the



**Fig. 4** Mid-span deflection at constant tendon eccentricity and varying prestressing force

cases, the progressive increment of prestressing force leads to a reduction in mid-span deflection. The higher opposing moment by the tendons (for increasing values of prestressing force) at particular eccentricity compensates the positive moment by the applied live load. Furthermore, there is a reduction in the mid-span deflection due to increased prestressing beam stiffness [21].



**Fig. 5** Predicted mid-span deflections plotted against numerical results

## 4 Statistical Analysis

Multivariable regression analysis is a statistical method for estimating the relationships between the variables. It includes the development of a relationship between a dependent variable and several independent variables or predictors. In this section, the results of the parametric study were analysed using a Microsoft Excel spreadsheet. The developed linear regression model for predicting the mid-span deflection ( $\delta$  in mm) of a continuous prestressed steel beam is expressed as follows:

$$\delta = 1.0238 + 0.0416(l) - 0.0176(e) - 0.0043(f) \quad (1)$$

where  $e$  (mm) is the eccentricity of prestressing tendons,  $l$  (kN) is applied load and  $f$  (kN) prestressing force. The coefficient of determination ( $R^2$ ) value for developed regression equations was 0.99, suggesting higher confidence levels. To validate the proposed model, the predicted deflection is compared with the deflection data set obtained from numerical analysis, as shown in Fig. 5. It can be observed in Fig. 5 that the results are closer to the similarity line, indicating the reliability of the proposed model.

## 5 Discussion and Conclusion

In the present paper, the feasibility of an external post-tensioning technique in improving the flexural capacity of the continuous steel beams has been examined

numerically. The analysis results show that the post-tensioning technique is a feasible approach for improving the flexural capacity of the continuous steel beam bridge. Significant reduction in mid-span deflection is observed for beams strengthened with tendons at higher eccentricity and prestressing force. Parametric study results show that higher eccentricity of tendons and prestressing force results in 15–27% improvement in the flexural capacity of the beam. An increase in prestressing force reduces mid-span deflection, but the effect is getting prominent at larger tendons eccentricity.

Furthermore, the simplified mathematical equation for predicting the mid-span deflection of continuous prestressed steel beam is proposed with a high  $R^2$  value. Overall, it can be concluded that the post-tensioning technique can be applied for repairing and rehabilitating existing heritage steel bridges or constructing new bridges with lighter sections and foundations. In future, the research work can be extended to the parametric investigation of continuous beams with different draped cable profiles, behaviour under cyclic loading, beams with unsymmetrical cross-section, etc.

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