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Simplified Thermoelectric Cooler (TEC) with Heatsinks Modeling and Simulation using Matlab and Simulink based-on Dimensional Analysis

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Abstract:
Energy sustainability is becoming central nowadays, especially in South Africa due to electricity blackouts. This article embarks on clean alternative energy based-on thermoelectricity with focus on thermoelectric cooler (TEC). However, in practice, a TEC requires heatsinks or heat-exchangers to function properly but heatsinks also introduce thermal resistance which affects its cooling power and efficiency − which must be addressed. As a result, a TEC with heatsinks model based-on dimensional analysis using Matlab / Simulink is examined. The research novel contributions are i) derivation of analytical forms for the TEC dimensionless cold and hot sides temperature and ii) Simulink TEC with heatsinks theoretical model based-on a simplified dimensional analysis, whereby a TEC with heatsinks parameters of interest can be easily simulated to ascertain analytically, numerically and graphically its optimal performance before implementing physically.

Keywords:
Alternative Energy, Dimensional Analysis, TEC with Heatsinks and Thermoelectricity.

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I. INTRODUCTION

According to [1], South Africa is being committed towards its fight for a low carbon and green economy with focus on energy mix to ensure energy security. As per ratification of the Paris Agreement, renewable energy and energy efficiency integration, as well as the policies and regulatory developments are progressively refined with respect to the present framework − which is hampered by ongoing electrical power-cuts due to high pressure on the national grid from increase demands (especially during winter) that the Eskom supply cannot sustain because of inadequate infrastructure that's failing from time to time. As a result, there is need for renewable energy to sustain the unstable national grid and also for private use. These developments necessitate our research for an alternative energy based on thermoelectricity with focus on basic households energy supply and energy efficient loads. Simply, thermoelectric generators (TEGs) cleanly convert low grade heat to DC electricity, whereas TECs enable clean production of cold and heat from electricity. Both TEGs and TECs if designed properly, can be fairly energy efficient and very handy for primary households energy needs such as lighting, cooling and heating that are basic necessities. An ideal theoretical framework for TEGs and TECs were respectively comprehensively presented in [2][3]; however, they lack practical considerations which this article seeks to bring forth and is thus, engaged next.

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II. BACKGROUND

Thermoelectricity or thermoelectric (TE) devices in the forms of TEGs and TECs, cannot practically and reliably function without heatsinks or some sorts of proper heat exchangers to maintain a safe temperature difference on the TEG and TEC sides and most importantly to dissipate the internal heat cause by Ohmic heating as a result of the current flowing through the TEG / TEC. This heat if not handled properly and becomes excessive, will not only cause inefficiency but can damage the TEG and TEC due to over heating, which will cause the solder joints binding the thermoelectric devices p-n thermocouples to melt. However, the addition of heatsinks in this case to address the thermoelectric devices practical limitations, also add / increases the thermal resistance on the TEGs and TECs sides, thus making the thermoelectric devices inefficient.

Asserted in [4]-[12], is a technique called dimensional analysis and can be used to improve TE devices efficiency when used with heatsinks. This article with focus on TEC, applies and developed further the apt approach in [4]; by using now a simplified Matlab / Simulink implementation. Presented next is the applicable mathematics that applies to the heatsinks when used with a TEC and the newly derived analytical formulas as well as the extra equivalent numerical simplifications we contributed, considering the prior analytical method limitation. This is then proceeded with the depiction of the simulator friendly user interface for the implemented TEC model using Matlab / Simulink, followed by the results and results interpretations as well as validations and finally, concluding remarks are drawn.

III. TEC with Heatsinks Applicable Maths

The applicable mathematics for a TEC when used with heatsinks is expressed and then followed by our additional simplification to the excellent approach presented by [4]. Dimensional analysis enables same unit parameters to be normalised between their minimum and maximum values, hence making them dimensionless and easy to work with. However, unique in [4] is the conversion of TEC heatsink thermal resistances to their convection conductance − this optimization method eliminates the TEC heatsink thermal resistances which are difficult to work with, in favour of the TEC fluid convection conductance which are relatable to the TEC heatsinks fluid temperature and are practically easier to work with. The maths analysis is presented thus:

i. TEC with Heatsinks General Heat Flow Equations

The following heat equations are expressed and their thermal resistance, thermal conductance and convection conductance relationships are established as follows:

$$Q_1 = K_1(T_{i1} - T_1) \quad \text{(W)} \quad (1)$$

$$Q_2 = K_2(T_2 - T_{i2}) \quad \text{(W)} \quad (2)$$

where Q_1 and Q_2 are respectively the heat flows on the heatsinks on the TEC cold-side and hot-side, K_1 and K_2 are the respective thermal conductance of the heatsinks on the TEC cold-side and hot-side, T_{i1} and T_{i2} are the respective temperatures of the heatsinks fluid on the TEC cold and hot sides and finally T_1 and T_2 are the respective cold and hot p-n junction temperatures on the TEC sides.

It should be noted that thermal resistance is the inverse of thermal conductance which is equivalent to convection conductance and therefore equations (1) and (2) in terms of the convection conductance (ηhA), can be written as:

$$Q_1 = \eta_1 h_1 A_1(T_{i1} - T_1) \quad \text{(W)} \quad (3)$$

$$Q_2 = \eta_2 h_2 A_2(T_2 - T_{i2}) \quad \text{(W)} \quad (4)$$

where η_1 is heatsink1 (on TEC cold-side) fin efficiency, h_1 is heatsink1 convection coefficient and A_1 is heatsink1 total surface area. Likewise, η_2 is heatsink2 (on TEC hot-side) fin efficiency, h_2 is heatsink2 convection coefficient and finally A_2 is heatsink2 total surface area.

TEC standard theoretical heat equations are given as:

$$Q_C = n[(SIT_1) - (K\Delta T)] - 0.5I^2R \quad \text{(W)} \quad (5)$$

$$Q_h = n[(SIT_2) - (K\Delta T)] + 0.5I^2R \quad \text{(W)} \quad (6)$$

where Q_c is the cooling power or heat absorbed on TEC cold-side, n quantity of thermocouples used in the TEC, S is the Seebeck coefficient, I the input current to the TEC, K is the thermal conductance (calculated as ak/l, where a is the area of the TEC p-n junction thermocouple, k the p-n junction thermocouple thermal conductivity and l the length of the TEC p-n junction thermocouple), ΔT is the temperature difference $T_2 - T_1$ across the TEC, $R=nr$ is the TEC module resistance, where r is the resistance of the TEC p-n junction thermocouple and Q_h is the heat emitted on TEC hot-side. It should be noted that the three terms on the right side of equations (5) and (6) are respectively the Seebeck / Peltier, Fourier / Heat and Ohmic / Joule terms with S, K and R considered as temperature independent. Considering the energy balance of the TEC with heatsinks system, equations (1), (3) and (5) are now respectively equivalent to equations (2), (4) and (6) which reduce to (with the unknown parameters being only T_1, T_2 and I):

$$Q_1 = Q_C = K_1(T_{i1} - T_1) = \eta_1 h_1 A_1(T_{i1} - T_1) = n[(SIT_1) - (K\Delta T)] - 0.5I^2R \quad \text{(W)} \quad (7)$$

$$Q_2 = Q_h = K_2(T_2 - T_{i2}) = \eta_2 h_2 A_2(T_2 - T_{i2}) = n[(SIT_2) - (K\Delta T)] + 0.5I^2R \quad \text{(W)} \quad (8)$$

Now, according to [4], to optimize the TEC with heatsinks (HS), the following dimensionless parameters are defined with respect to fluid 2 (air or water on TEC hot-side heatsink2) and since the optimization is made dimensionless with respect to fluid 2, fluid 2 convection conductance and temperature (T_{i2}) are initially provided.

ii. TEC-HS Dimensionless Thermal Conductance (N_k)

This is the thermal conductance K divided by the convection conductance ηhA in fluid 2, expressed as:

$$N_k = K / \eta hA = (ak/l) / \eta_2 h_2 A_2 \quad (9)$$

iii. TEC-HS Dimensionless Convection (N_h)

This is fluid 1 convection conductance divided by fluid 2 convection conductance, expressed as.

$$N_h = \eta_1 h_1 A_1 / \eta_2 h_2 A_2 \quad (10)$$

iv. TEC-HS Dimensionless Current (N_i)

$$N_i = SI / K = SI / (ak/l) \quad (11)$$

v. TEC Dimensionless Temperatures (T_{s1}, T_{s2}, T_i)

The dimensionless temperatures are given as:

T_1 dimensionless temperature: $T_{s1} = T_1 / T_{i2}$ (12)

T_2 dimensionless temperature: $T_{s2} = T_2 / T_{i2}$ (13)

Fluids dimensionless temperature: $T_i = T_{i1} / T_{i2}$ (14)

vi. TEC-HS Dimensionless Cooling Power (Q_{s1})

$$Q_{s1} = Q_1 / \eta_2 h_2 A_2 T_{i2} \quad (15)$$

vii. TEC-HS Dimensionless Heating Power (Q_{s2})

$$Q_{s2} = Q_2 / \eta_2 h_2 A_2 T_{i2} \quad (16)$$

NB: Q_{s2} is also known as the dimensionless heat released by the heatsink on the TEC hot-side.

viii. TEC Dimensionless Input Power (P_{ins} *or* W_s)

$$P_{ins} = P_{in} / \eta_2 h_2 A_2 T_{i2} \quad (17)$$
$$W_s = P_{ins} = Q_{s2} - Q_{s1} \quad (18)$$

ix. TEC Dimensionless Input Voltage (V_{ins})

$$V_{ins} = V / nST_{i2} \quad (19)$$
$$V_{ins} = P_{ins} / N_k N_i \quad (20)$$

x. TEC *Coefficient of Performance (CoP)*

NB: *CoP* is already dimensionless.

$$CoP = Q_{s1} / W_s \quad (21)$$

xi. TEC-HS Dimensionless Cooling Power (Q_{s1}) in terms of T_{s1}

$$Q_{s1} = N_h (T_i - T_{s1}) \quad (22)$$

xii. TEC-HS Dimensionless Heating Power (Q_{s2}) in terms of T_{s2}

$$Q_{s2} = T_{s2} - 1 \quad (23)$$

According to [4], by using the dimensionless equations (9) - (14), equations (3) - (6) boils down to the following two equations (24) and (25) with five unknowns which must be solved for T_{s1} and T_{s2} in terms of five independent dimensionless parameters of N_k, N_h, N_i, T_i and ZT_{i2}.

$$N_h(T_i - T_{s1})/N_k = N_i T_{s1} - ((Ni^2)/(2ZT_{i2})) + (T_{s1} - T_{s2}) \quad (24)$$
$$(T_{s2} - 1)/N_k = N_i T_{s2} + ((Ni^2) / (2ZT_{i2})) + (T_{s1} - T_{s2}) \quad (25)$$

These two expressions (i.e. equations (24) and (25)) and the whole process is very cumbersome to determine T_{s1} and T_{s2}; as a result, it was not solved further in [4] due to limitation in the analytical method and were expressed as shown in equations (26) and (27). A numerical method using tables/graphs/approximations were employed by [4] and also a simulation using Mathcad was recommended.

$$T_{s1} = f(N_k, N_h, N_i, T_i, ZT_{i2}) \quad (26)$$
$$T_{s2} = f(N_k, N_h, N_i, T_i, ZT_{i2}) \quad (27)$$

However, we analytically simplify equations (24) and (25) further and expressed T_{s1} and T_{s2} directly in terms of the five independent dimensionless parameters N_k, N_h, N_i, T_i and ZT_{i2}. We developed the numeric approximation method further to clearly prove and calculate the exact values for T_{s1} and T_{s2} by using the now newly introduced analytical expressions for T_{s1} and T_{s2} – which aren't found in the available literature to the best of our knowledge. In the next section, our approach is developed further from equations (24) and (25) to solve for T_{s1} and T_{s2} directly.

xiii. TEC Dimensionless Temperatures (T_{s1}, T_{s2}) Formula

Subtracting equation (24) from (25) and simplifying further gives equation (28) as expressed below:

$$(T_{s2} - 1) / N_k - (N_h (T_i - T_{s1})) / N_k = N_i T_{s2} - N_i T_{s1} + N_i^2 / ZT_{i2} \quad (28)$$

Solving for T_{s1} and T_{s2} yield equations (29) and (30).

$$T_{s1} = (2N_k ZT_{i2} + Ni^2 N_k + 2N_i^2 N_k^2 - N_i^3 N_k^2 + 2N_h T_i ZT_{i2} + 2N_h N_k T_i ZT_{i2} - 2N_h N_i N_k T_i ZT_{i2}) / (2ZT_{i2}(N_h + N_k - N_i^2 N_k^2 + N_h N_k + N_i N_k - N_h N_i N_k)) \quad (29)$$
$$T_{s2} = (2N_h ZT_{i2} + 2N_k ZT_{i2} + 2N_i^2 N_k^2 + N_i^3 N_k^2 + 2N_i N_k ZT_{i2} + N_h N_i^2 N_k + 2N_h N_k T_i ZT_{i2}) / (2ZT_{i2}(N_h + N_k - N_i^2 N_k^2 + N_h N_k + N_i N_k - N_h N_i N_k)) \quad (30)$$

As can be seen, equations (29) and (30) for T_{s1} and T_{s2}, are novel analytical formulas derived in terms of N_k, N_h, N_i, T_i and ZT_{i2}; which together with equation (28) can now be solved, given the N_k, N_h, N_i, T_i and ZT_{i2} values as follows.

Let ZT_{i2}=1 and N_h=1; equations (28) - (30) reduce to:

$$(T_{s2} - 1)/N_k - (T_i - T_{s1})/N_k = N_i T_{s2} - N_i T_{s1} + N_i^2 \quad (31)$$
$$T_{s1} = (2N_k + 2T_i + 2N_k T_i + N_i^2 N_k + 2N_i^2 N_k^2 - N_i^3 N_k^2 - 2N_i N_k T_i) / (-2N_i^2 N_k^2 + 4N_k + 2) \quad (32)$$
$$T_{s2} = (2N_k + 2N_k T_i + N_i^2 N_k + 2N_i^2 N_k^2 + N_i^3 N_k^2 + 2N_i N_k + 2) / (-2N_i^2 N_k^2 + 4N_k + 2) \quad (33)$$

Let ZT_{i2}=1, N_h=1 and T_i=0.97; (28) - (30) become:

$$(T_{s2} - 1)/N_k + (T_{s1} - 97/100)/N_k = N_i T_{s2} - N_i T_{s1} + N_i^2 \quad (34)$$
$$T_{s1} = (-N_i^3 N_k^2 + 2N_i^2 N_k^2 + N_i^2 N_k - (97N_i N_k)/50 + (197N_k)/50 + 97/50) / (-2N_i^2 N_k^2 + 4N_k + 2) \quad (35)$$
$$T_{s2} = (N_i^3 N_k^2 + 2N_i^2 N_k^2 + N_i^2 N_k + 2N_i N_k + (197N_k)/50 + 2) / (-2N_i^2 N_k^2 + 4N_k + 2) \quad (36)$$

Let ZT_{i2}=1, N_h=1, T_i=0.97 and N_k=0.3; (28) - (30) are:

$$(10T_{s1})/3 + (10T_{s2})/3 - 197/30 = N_i T_{s2} - N_i T_{s1} + N_i^2 \quad (37)$$
$$T_{s1} = ((9N_i^3)/100 - (12N_i^2)/25 + (291N_i)/500 - 1561/500) / ((9N_i^2)/50 - 16/5) \quad (38)$$
$$T_{s2} = -((9N_i^3)/100 + (12N_i^2)/25 + (3N_i)/5 + 1591/500) / ((9N_i^2)/50 - 16/5) \quad (39)$$

Finally, let ZT_{i2}=1, N_h=1, T_i=0.97, N_k=0.3 and Ni=0.5; equations (28) - (30) give the final optimum solutions as:

$$T_{s1} = 409 / 230 - (17T_{s2}) / 23 \quad (40)$$
$$T_{s1} = 0.9318$$
$$T_{s2} = 1.1452$$

Equation (40) is the simplified optimal link between T_{s1} and T_{s2} – which both can be put in equations (22) and (23) to compute Q_{s1} and Q_{s2} and finally compute W_s using (18). Can then further find CoP, V_{ins}, V, P_{in}, Q_2, Q_1, T_2, T_1 and I.

IV. TEC WITH HEATSINKS MODELING

The applicable TEC heatsinks mathematics based on dimensional analysis was presented in Section III and the equations for T_{s1} and T_{s2} were developed and analytically expressed in terms of the five independent parameters N_k, N_h, N_i, T_i and ZT_{i2} and furthermore optimally simplified and validated in terms of T_{s1} and T_{s2} using equation (40). As evident, the whole analytical and numerical processes could be very cumbersome as well as erroneous if done manually. In light of this, a TEC model with heatsinks on both the TEC cold and hot sides, was designed based on the equations presented in Section III and modeled using Matlab and Simulink. Included in the simulated TEC with heatsinks model, are some of the i) TEC parameters analysed in Section III including: the Seebeck coefficient *S*, the absolute temperature in Kelvin 273.15, heatsink1 fluid temperature T_{i1}, heatsink2 fluid temperature T_{i2}, if need be infinite TECs in series T_s and or parallel T_p, TEC p-n thermocouple resistance *r*, thermal conductivity *k*, electrical resistivity *Ƥ*, p-n thermocouple area *a*, p-n thermocouple length *l* and the TEC total cooling power required *Qreq*, and ii) heatsinks parameters such as the area of heatsink base *Ab*, heatsink1 fins efficiency n_1, heatsink1 fluid convection coefficient h_1, heatsink1 total area A_1 including fins sides, heatsink2 fins efficiency n_2, heatsink2 fluid convection coefficient h_2 and heatsink2 total area A_2 including fins sides. All these parameters can simply be entered and the TEC optimal parameters value calculated as portrayed in Fig. 1. Optionally, the TEC dimensionless parameters such as dimensionless current N_i, dimensionless thermal conductance N_k, the coefficient of performance *CoP*, the dimensionless cooling power Q_{s1}, TEC cold-side dimensionless temperature T_{s1}, TEC hot-side dimensionless temperature T_{s2} and the dimensionless voltage N_v as well as the TEC half dimensionless optimal parameters not discussed (which are simply half the value of the dimensionless parameters discussed, to ensure safer design margins), can be either input manually (as shown on the left side of the TEC with heatsinks in Fig. 1) or automatically calculated and used to finally compute the actual physical parameters values in SI units, as depicted on the right side of the TEC with heatsinks in Fig. 1. Some of the TEC physical parameters of interest auto calculated include the TEC figure of merit *Z*, the TEC number of p-n thermocouples *n*, the TEC cooling power Q_1, the TEC input power W_n or P_{in}, the TEC cold-side temperature T_1, the TEC hot-side temperature T_2, the TEC input voltage V_{in}, the TEC input current *I*, the total number of TEC modules required *N*, TEC total power consumption W_t, the TEC total internal source resistance *R*, the TEC temperature difference *DT* or ΔT, the TEC maximum *CoP* (CoP_{max}), the input current I_{cop} needed to give *CoP*, the TEC with heatsinks cooling power density *QPD*, fluids 1 and 2 dimensionless convection conductance (*nhA*) and as well the TEC corresponding half optimal values for the mentioned parameters. The input and output parameters were grouped together based on their similarities, named and colour coded accordingly to make the TEC with heatsinks model easy to understand and also user friendly. In addition to the TEC with heatsinks numeric model calculations, characteristics curves of some of the crucial parameters such as the TEC dimensionless cooling power Q_{s1}, the TEC dimensionless heating power Q_{s2}, the TEC dimensionless input power P_{ins} or W_s, the TEC *CoP* and the TEC dimensionless temperatures T_{s1} and T_{s2}; were plotted against the TEC dimensionless input current N_i and the TEC dimensionless thermal conductance N_k to graphically calculate Q_{s1}, Q_{s2}, P_{ins}, *CoP*, T_{s1} and T_{s2} optimal values for i) ZT_{i2}=1, N_h=1, T_i =0.97 N_k=0.1-1 and N_i=0-1 and ii) for ZT_{i2}=1, N_h=1, T_i =0.97 N_k=0-1 and N_i =0.1-1. These graphical results are displayed next in Figs. 2 to 13.

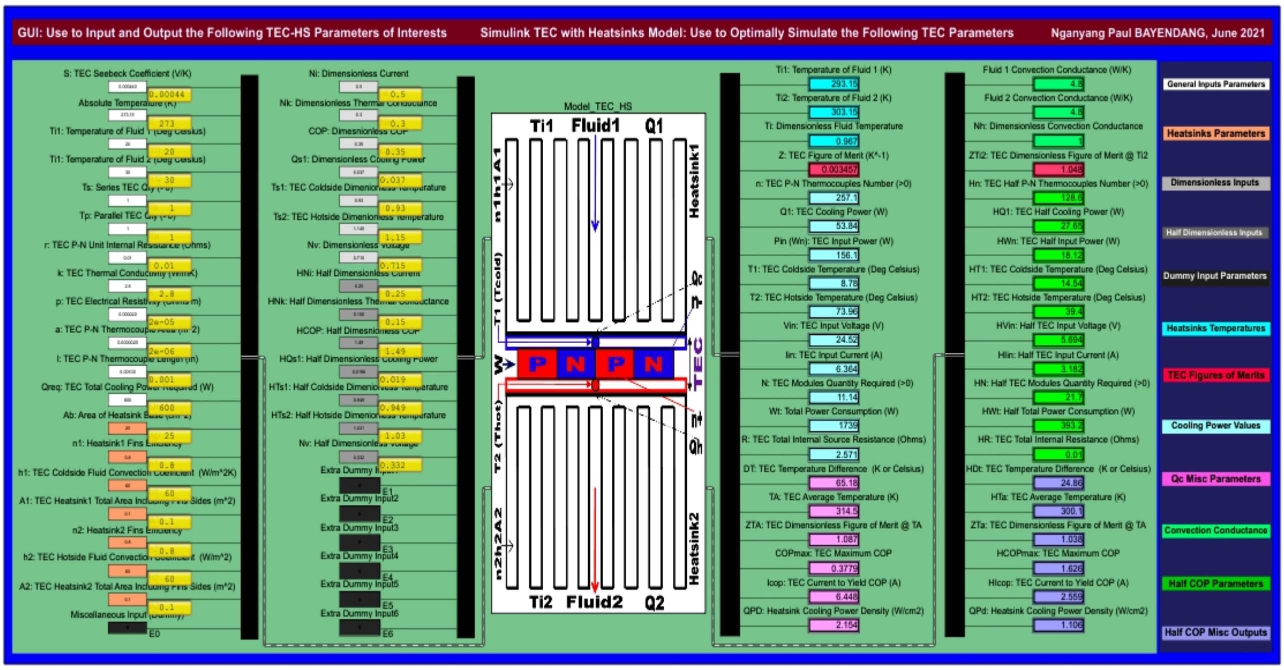

Fig. 1. Novel Matlab/Simulink TEC with Heatsinks Model: Simulated Numerical Results

V. TEC with Heatsinks Graphical Results

The following characteristic curves were plotted:

A. With ZT_{i2}=1, N_h=1, T_i =0.97 N_k=0.1-1 and N_i=0-1

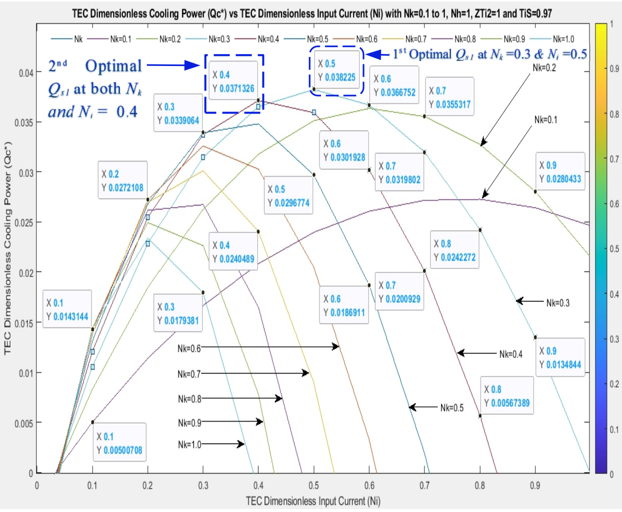

Fig. 2. Dimensionless: Cooling Power Q_{s1} *vs* Input Current N_i

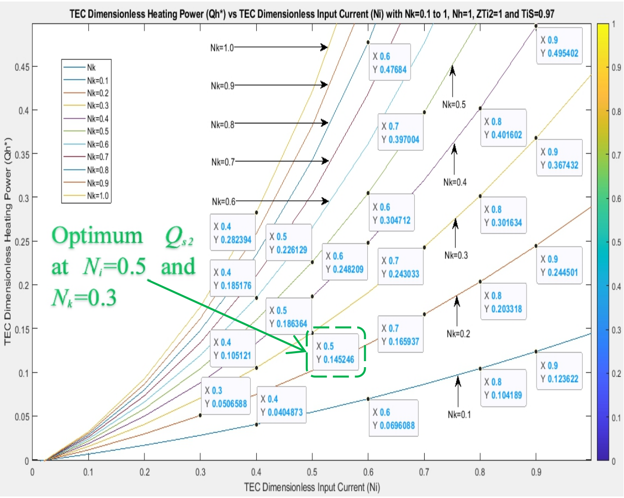

Fig. 3. Dimensionless: Heating Power Q_{s2} *vs* Input Current N_i

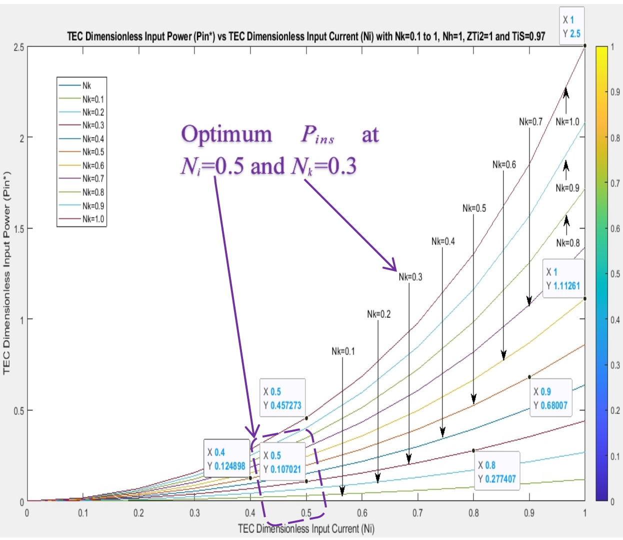

Fig. 4. Dimensionless: Input Power P_{ins} *vs* Input Current N_i

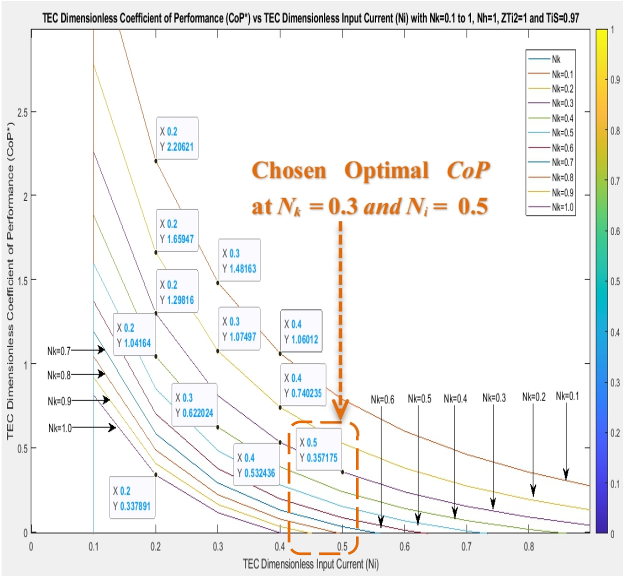

Fig. 5. Dimensionless: *CoP vs* Input Current N_i

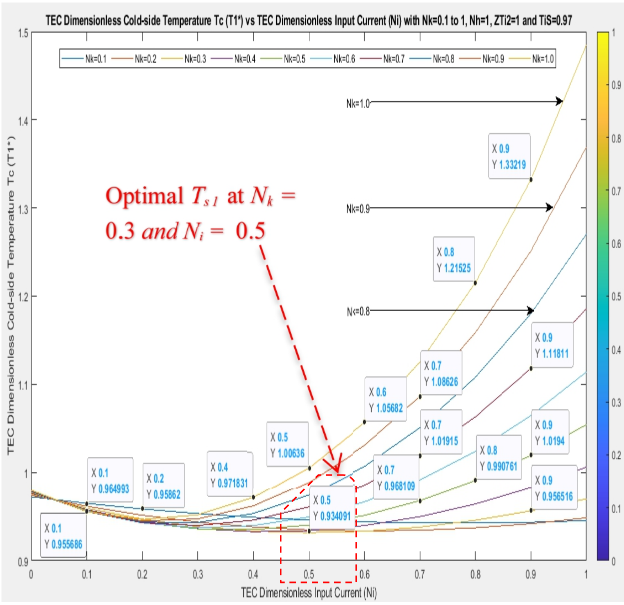

Fig. 6. Dimensionless: TEC Cold-side T_{s1} *vs* Input Current N_i

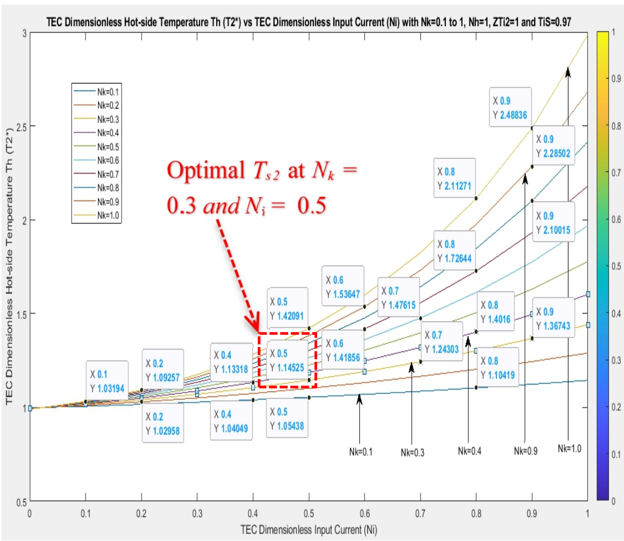

Fig. 7. Dimensionless: TEC Hot-side T_{s2} *vs* Input Current N_i

B. With ZT_{i2}=1, N_h=1, T_i =0.97 N_i=0.1-1 and N_k=0-1

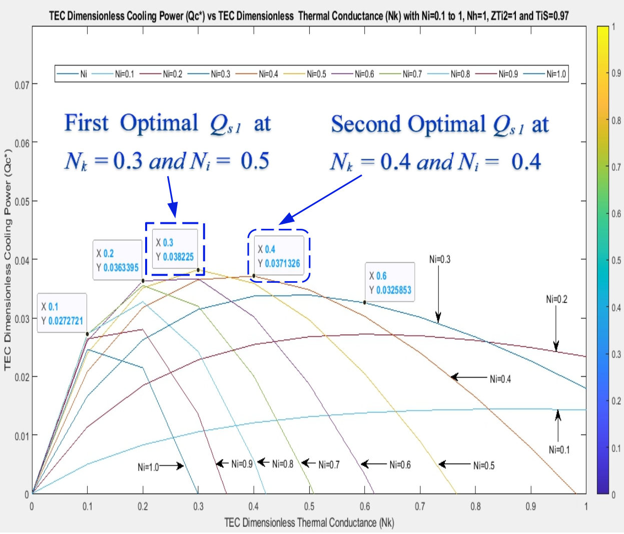

Fig. 8. Dimensionless: Cooling Power Q_{s1} *vs* Thermal Conductance N_k

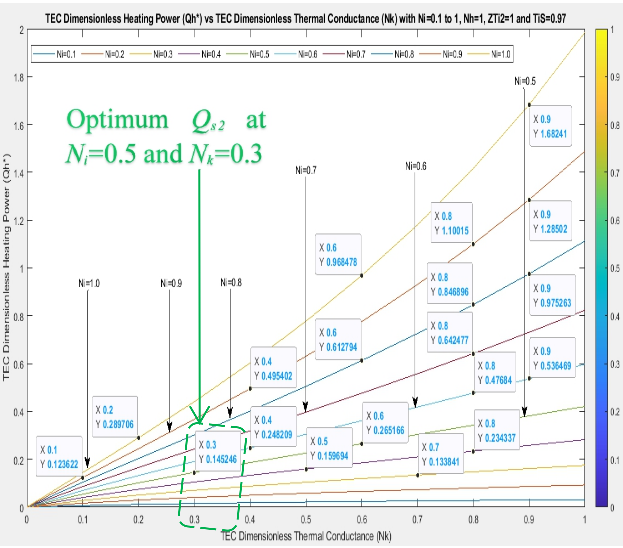

Fig. 9. Dimensionless: Heating Power Q_{s2} *vs* Thermal Conductance N_k

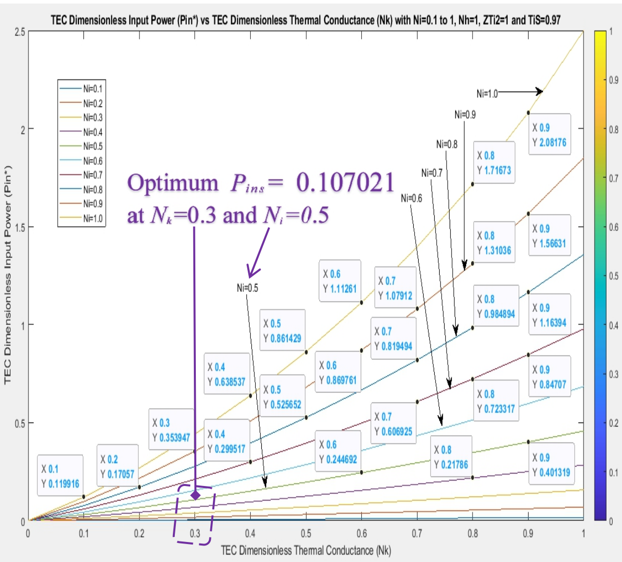

Fig. 10. Dimensionless: Input Power P_{ins} *vs* Thermal Conductance N_k

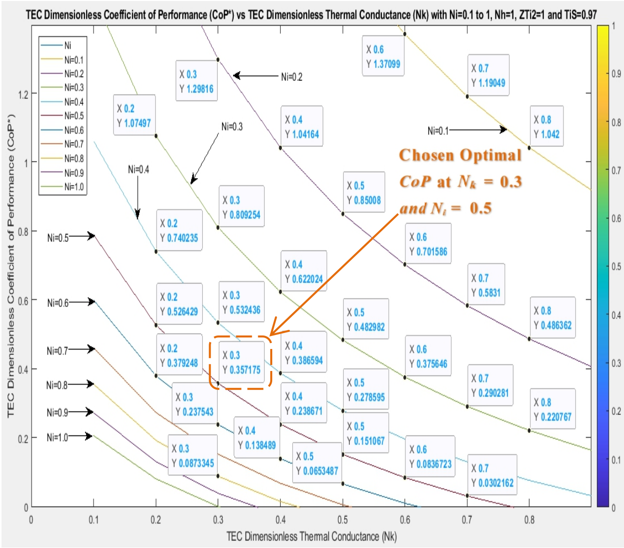

Fig. 11. Dimensionless: *CoP vs* Thermal Conductance N_k

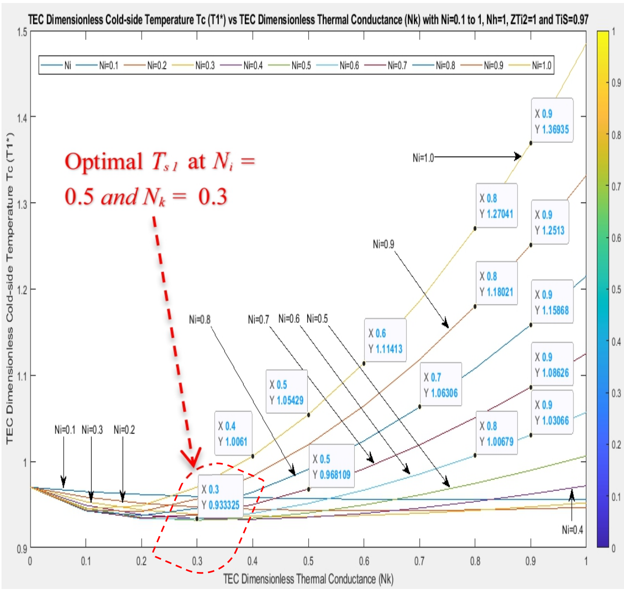

Fig. 12. Dimensionless: TEC Cold-side T_{s1} *vs* Thermal Conductance N_k

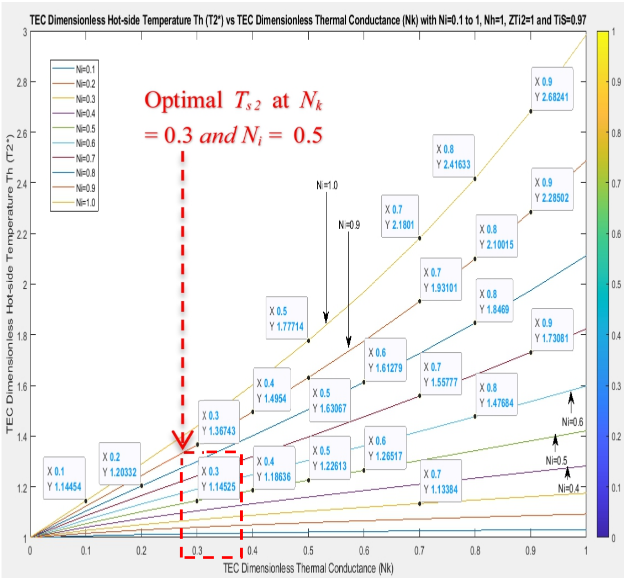

Fig. 13. Dimensionless: TEC Hot-side T_{s2} *vs* Thermal Conductance N_k

VI. Results Interpretation and Validation

In Section III, the analytical formulas for T_{s1} and T_{s2} were derived and were used to calculate T_{s1} and T_{s2} optimal values and in Section IV, a TEC with heatsinks simulated model was implemented and was employed to determine numerically its optimal results and as well graphically as portrayed in Section V. These results are discussed and validated herein with the theoretical results presented in [4]. Fig. 1 simulated results are validated with Fig. 14 results. For example, highlighted in red in Fig. 14, the T_{s1} and T_{s2} values are valid and approximately the same as analytically, numerically and graphically found in Figs. 1, 6, 7, 12 & 13. The practical relevance of T_{s1} and T_{s2} results is, it enables an optimal design by easily manipulating the temperatures of the heatsinks fluid if known and at optimal performance, T_{s1} and T_{s2} are minimum for N_i and N_k values from 0.3 - 0.5 as pictured and validated with Fig. 15, highlighted in red.

Input	$Q^*_{1,opt}$ (dimensionless)	$Q_{1,opt}$ (Actual)	$COP_{1/2opt}$ (dimensionless)	$COP_{1/2opt}$ (Actual)
$T_{\infty 1} = 20°C$, $T_{\infty 2} = 30°C$	$N_k = 0.3$	$n = 257$	$N_k = 0.15$	$n = 128$
$A = 2\ mm^2$, $L = 1\ mm$	$N_h = 1$	$\eta_1\ h_1\ A_1 =$ 4.8 W/K	$N_h = 1$	$\eta_1\ h_1\ A_1 = 4.8$ W/K
$\eta_2 = 0.8$, $h_2 = 60\ W/m^2K$,	$N_I = 0.5$	$I = 6.36$ A	$N_I = 0.25$	$I = 3.18$ A
$A_2 = 1000\ cm^2$	$T^*_\infty = 0.967$	$T_{\infty 1} = 20°C$	$T^*_\infty = 0.967$	$T_{\infty 1} = 20°C$
Base area $A_b = 5$ cm × 5 cm	$ZT_{\infty 2} = 1.0$	$ZT_{\infty 2} = 1.0$	$ZT_{\infty 2} = 1.0$	$ZT_{\infty 2} = 1.0$
$\alpha_p = -\alpha_n = 220\ \mu V/K$	$T^*_1 = 0.930$	$T_1 = 8.7°C$	$T^*_1 = 0.949$	$T_1 = 14.4°C$
$\rho_p = \rho_n = 1.0 \times 10^{-3}\ \Omega cm$	$T^*_2 = 1.145$	$T_2 = 73.9°C$	$T^*_2 = 1.031$	$T_2 = 39.4°C$
$k_p = k_n = 1.4 \times 10^{-2}$ W/cmK	$Q^*_1 = 0.037$	$Q_1 = 54.4$ W	$Q^*_1 = 0.019$	$Q_1 = 26.9$ W
$(Z = 3.457 \times 10^{-3}\ K^{-1})$		$COP = 0.35$		$COP = 1.49$
(R =0.01Ω per thermocouple)	$N_V = 0.715$	$V_n = 24.5$ V	$N_V = 0.332$	$V_n = 5.7$ V
	$T^*_{\infty,max} = 1.2$	$T_{\infty 1,max} =$ 91.3°C	$T^*_{\infty,max} = 1.06$	$T_{\infty 1,max} =$ 49.7°C
	$T^*_{\infty,min} = 0.83$	$T_{\infty 1,min} =$ −21.8°C	$T^*_{\infty,min} = 0.84$	$T_{\infty 1,min} =$ −19.2°C
(Cooling Power Density $P_d = Q_1/A_b$)	-	$P_d = 2.18$ W/cm²	-	$P_d = 1.08$ W/cm²

Fig. 14. TEC with HS Optimal Results Validator (adapted from [4])

Figs. 2 and 8 depict the TEC optimal dimensionless cooling power $Q_{s1} = 0.038$; respectively at N_i and N_k values of 0.5 and 0.3. This is validated with Figs. 14, 15, 16 and 17 shown highlighted in blue. However, at both N_i and N_k values of 0.4 in Figs. 2 and 8, we reckon there also exists an optimal value of Q_{s1}=~0.037; since it's optimum, if also a similar Q_{s1} is had with less N_i and more N_k. So, depending on the available resources, two optimum Q_{s1} points exist.

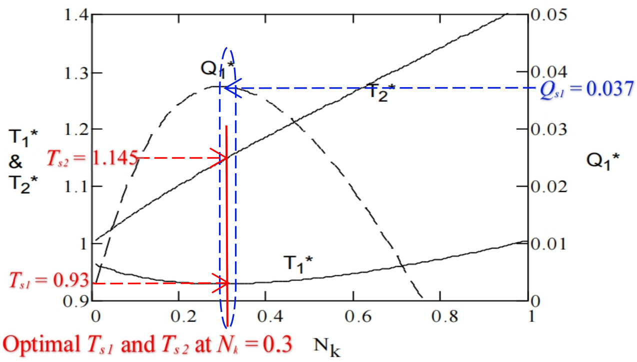

Fig. 15. TEC with HS Q_{s1}, T_{s1} and T_{s2} Optimal Results (adapted from [4])

In Figs. 3 and 9, optimum value of Q_{s2} = 0.145246 is found when N_i = 0.5 and N_k=0.3 as highlighted in green. However, increasing N_k, gives more Q_{s2} at the same N_i=0.5. The practical gist of this is opting for a cooler or a heater.

Figs. 4 and 10 show P_{ins}, which is directly proportional to N_i and N_k. Purple highlighted is optimal P_{ins} = 0.107021 at N_k= 0.3 and N_i= 0.5. This is also validated with Fig. 16.

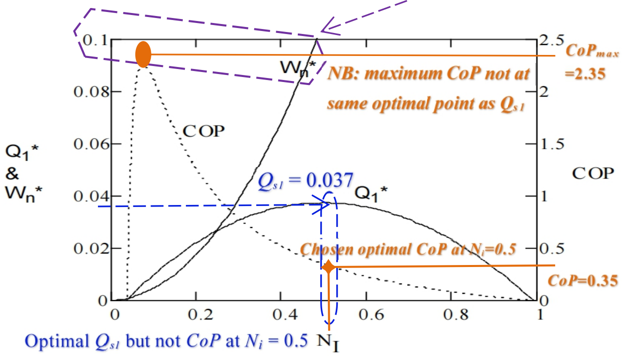

Fig. 16. TEC-HS W_s, Q_{s1} and CoP Optimal Results (adapted from [4])

Finally, the CoP is shown in Figs. 5 and 11; however, optimal CoP as evident in Figs. 5, 11, 16 and 17, is not attained at the optimal Q_{s1} using the dimensional analysis. As a result, the optimal CoP of 0.35 is manually chosen at optimal N_i = 0.5 and N_k=0.3 which is highlighted in orange in Figs. 5 and 11 and validated using Figs. 14, 16 and 17.

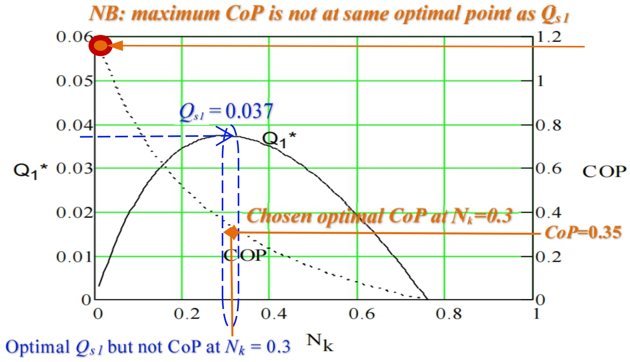

Fig. 17. TEC with HS Q_{s1} and CoP Optimal Results (adapted from [4])

VII. Conclusion

This study began by briefly highlighting the ongoing South Africa energy crisis and the need for renewable energy/power sources and as well energy efficient loads. Alternative energy based on thermoelectricity and with focus on TEC was investigated to determine optimal TEC design with heatsinks when considering a good practical implementation. However, heatsinks introduce thermal resistances which negatively affect the TEC performance. Dimensional analysis, in particular one that easily converts TEC thermal conductance to convection conductance was researched as the optimisation tact to facilitate an optimal practical design. This approach was developed and new analytical formulas were introduced to compute the TEC optimal dimensionless cold and hot sides temperatures. These formulas were verified using analytical, numerical and graphical solutions with the aid of an easy TEC with heatsinks model implemented using Matlab and Simulink. The optimal results were validated with a prior research using TEC manufacturers typical parameters value used in the industry. These outcomes define the research impacts, from which a physical design can be done and the new TEC model benchmark with for further use to devise and implement a clean energy efficient cooling/heating device.

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