Hindawi
International Journal of Mathematics and Mathematical Sciences
Volume 2019, Article ID 9382525, 17 pages
https://doi.org/10.1155/2019/9382525

Research Article

Fischer-Clifford Matrices and Character Table of the Maximal Subgroup $(2^9 : (L_3(4)) : 2$ of $U_6(2) : 2$

Abraham Love Prins [1] **and Ramotjaki Lucky Monaledi** [2]

[1] *Department of Mathematics and Physics, Faculty of Applied Sciences, Cape Peninsula University of Technology, P.O. Box 1906, Bellville 7535, South Africa*
[2] *Department of Mathematics, Faculty of Military Science, Stellenbosch University, Private Bag X2, Saldanha 7395, South Africa*

Correspondence should be addressed to Abraham Love Prins; prinsab@cput.ac.za

Received 11 October 2018; Accepted 26 December 2018; Published 25 February 2019

Academic Editor: Adolfo Ballester-Bolinches

The automorphism group $U_6(2) : 2$ of the unitary group $U_6(2) \cong Fi_{21}$ has a maximal subgroup $\overline{G}$ of the form $(2^9 : (L_3(4)) : 2$ of order 20643840. In this paper, Fischer-Clifford theory is applied to the split extension group $\overline{G}$ to construct its character table. Also, class fusion from $\overline{G}$ into the parent group $U_6(2) : 2$ is determined.

1. Introduction

The unitary simple group $U = U_6(2)$ has outer automorphisms of orders 2, 3, and 6 and hence automorphism groups of the forms $U_1 = U_6(2) : 2$, $U_2 = U_6(2) : 3$, and $U_3 = U_6(2) : S_3$ exist for U (see the ATLAS [1]). The reader is referred to [2] for more information about the construction of matrix representations for the covering and automorphism groups of $U_6(2)$. Recently in [3], a 3-local identification is given for the group $PSU_6 \cong U_6(2)$ and its automorphism groups $PSU_6(2) : 3$, $PSU_6(2) : 2$, and $PSU_6(2) : S_3$. Also, we found in the ATLAS that one of the 16 maximal subgroups of U is a split extension group A of the type $2^9 : L_3(4)$ of index 891 and has order 10321920. The group A has automorphism groups of the form $A_1 = (2^9 : L_3(4)) : 2$, $A_2 = (2^9 : L_3(4)) : 3$, and $A_3 = (2^9 : L_3(4)) : S_3$ which sit maximally inside the groups U_1, U_2, and U_3, respectively.

The character table of A is stored in the GAP Library [4], whereas the character table of A_1 and A_2 are not yet uploaded in GAP. In this paper, the unknown Fischer-Clifford matrices [5] of A_1 and its associated character table will be constructed. Readers are referred to [6] on a survey of Fischer-Clifford theory. It is interesting to note that our group A_1 is the stabilizer $\mathrm{Stab}_{U_1}(v_1)$ of a singular vector $v_1 \in V_{20}(2)$, where $V_{20}(2)$ is the irreducible module of dimension 20 over the field $GF(2)$ for $U_6(2) : 2$ (see [3]). The method of coset analysis as discussed in [7–9] will be used in the computation of the conjugacy classes of elements of the group A_1. The Fischer-Clifford matrices and character table of A_3 were determined in [10].

2. Group 2^9:($L_3(4)$:2)

In this section, using a suitable permutation representation of $U_6(2) : 2$, we identify our group $A_1 = (2^9 : L_3(4)) : 2$ as a split extension 2^9 by $L_3(4) : 2$ with the aid of GAP [4] and MAGMA [11]. Then with the help of MAGMA we represent $L_3(4) : 2$ as a matrix group G of degree 9 over the Galois field $GF(2)$. Since G acts absolutely irreducibly on its natural module 2^9, a split extension S of the form $2^9 : (L_3(4) : 2)$ exists. Then we create S as a subgroup of $SL_{10}(2)$ and show with the help of MAGMA that A_1 is indeed an isomorphic copy of S.

We construct $U_1 = U_6(2) : 2$ within GAP, using its smallest permutation representation of degree 672 found in the online ATLAS of Finite Group Representations [12].

Next, we use the GAP commands "MS:= ConjugacyClassesMaximalSubgroups (U_1)", "A1:=MS[4]", and "Size(A1)" to represent $A_1 = (2^9 : (L_3(4)) : 2$ as a permutation group on 672 points and then use this permutation representation to construct A_1 within MAGMA. Using the sequence of MAGMA commands "a,b:=ChiefSeries(A1)", "N:= a[3]", "NormalSubgroups(A1)", "IsNormal(A1,a[3])", "IsElementaryAbelian(N)", "C:= Complements(A1,N)", "Order(C[1])", "C[1] meet N", and "IsIsomorphic(C[1], $L_3(4) : 2$)", we verified that $A_1 = (2^9 : L_3(4)) : 2 \cong 2^9 : (L_3(4) : 2)$.

Having A_1 as a permutation group on 672 points, we use the MAGMA commands "M:=GModule(A1,N)" and "'M:Maximal" to represent $L_3(4) : 2$ as a matrix group G of degree 9 over the Galois field $GF(2)$. Thus we obtain the matrix group $G = \langle g_1, g_2 \rangle$ having 14 conjugacy classes of elements, where $o(g_1) = 4$ and $o(g_2) = 14$. The generators g_1 and g_2 of G are as follows:

$$g_1 = \begin{pmatrix} 1&0&0&0&0&1&1&1&0\\ 0&1&0&1&0&1&0&0&0\\ 1&1&0&1&0&0&0&0&0\\ 1&0&0&1&1&1&0&0&1\\ 0&0&1&1&1&1&1&1&1\\ 0&1&1&1&1&0&0&0&1\\ 0&1&0&0&1&0&0&0&0\\ 1&0&1&0&1&1&1&0&0\\ 0&1&0&0&1&0&1&0&1 \end{pmatrix}$$

$$g_2 = \begin{pmatrix} 1&0&0&1&1&0&0&1&1\\ 1&1&0&0&0&1&1&1&1\\ 0&0&0&0&0&1&1&1&0\\ 0&1&0&1&0&1&1&1&0\\ 1&0&1&0&1&1&0&0&0\\ 1&1&1&1&1&0&1&1&1\\ 0&1&1&1&0&1&1&1&1\\ 1&0&1&1&0&0&1&0&1\\ 0&1&0&1&0&1&1&0&0 \end{pmatrix}. \quad (1)$$

The MAGMA command "IsAbsolutelyIrreducible(G)" tells us that the action of the matrix group G on its natural module 2^9 is absolutely irreducible. Thus a split extension of the type $2^9 : (L_3(4) : 2)$ does exist. Hence we can construct $2^9 : (L_3(4) : 2)$ as a subgroup S of $SL_{10}(2)$ such that $S = \langle s_1, s_2, s_3 \rangle$ and $L_3(4) : 2 = \langle s_1, s_2 \rangle$, where $o(s_1) = 4$, $o(s_2) = 14$, and $o(s_3) = 2$. The generators of the matrix group S of degree 10 over $GF(2)$ are as follows:

$$s_1 = \begin{pmatrix} 1&0&0&0&0&1&1&1&0&0\\ 0&1&0&1&0&1&0&0&0&0\\ 1&1&0&1&0&0&0&0&0&0\\ 1&0&0&1&1&1&0&0&1&0\\ 0&0&1&1&1&1&1&1&1&0\\ 0&1&1&1&1&0&0&0&1&0\\ 0&1&0&0&1&0&0&0&0&0\\ 1&0&1&0&1&1&1&0&0&0\\ 0&1&0&0&1&0&1&0&1&0\\ 0&0&0&0&0&0&0&0&0&1 \end{pmatrix}$$

$$s_2 = \begin{pmatrix} 1&0&0&1&1&0&0&1&1&0\\ 1&1&0&0&0&1&1&1&1&0\\ 0&0&0&0&0&1&1&1&0&0\\ 0&1&0&1&0&1&1&1&0&0\\ 1&0&1&0&1&1&0&0&0&0\\ 1&1&1&1&1&0&1&1&1&0\\ 0&1&1&1&0&1&1&1&1&0\\ 1&0&1&1&0&0&1&0&1&0\\ 0&1&0&1&0&1&1&0&0&0\\ 0&0&0&0&0&0&0&0&0&1 \end{pmatrix} \quad (2)$$

$$s_3 = \begin{pmatrix} 1&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&1&0\\ 1&0&0&0&0&0&0&0&0&1 \end{pmatrix}.$$

Since we can represent S and A_1 as a matrix and permutation group, respectively, we use the MAGMA command "IsIsomorphic (S, A_1)" to confirm that $S \cong A_1$. Hence we can regard $A_1 = (2^9 : L_3(4)) : 2$ as the split extension $S = 2^9 : (L_3(4) : 2)$.

3. The Conjugacy Classes of $\overline{G}=2^9{:}(L_3(4){:}2)$

In this section, the conjugacy classes of $\overline{G}$ are computed using the technique of coset analysis and readers are encouraged to consult [8, 9] for a sound theoretical background on this technique.

Throughout the remainder of this chapter, let $\overline{G} = 2^9 : (L_3(4) : 2)$ be a split extension of $N = 2^9$ by $L_3(4) :$

Table 1: The values of $\chi(L_3(4):2 \mid 2^9)$ on the different classes of $L_3(4):2$.

$[h]_{L_3(4):2}$	$1A$	$2A$	$2B$	$3A$	$4A$	$4B$	$4C$	$5A$	$6A$	$7A$	$7B$	$8A$	$14A$	$14B$
$\chi(L_3(4):3 \mid P_1)$	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$\chi(L_3(4):3 \mid P_2)$	21	7	5	3	1	1	3	1	1	0	0	1	0	0
$\chi(L_3(4):3 \mid P_3)$	210	28	18	3	2	2	4	0	1	0	0	0	0	0
$\chi(L_3(4):3 \mid P_4)$	280	28	8	1	4	4	0	0	1	0	0	2	0	0
k	512	64	32	8	8	8	8	2	4	1	1	4	1	1

2, where N is the vector space $V_9(2)$ of dimension 9 over $GF(2)$ on which the linear group G acts. Since $G = \langle g_1, g_2\rangle$ is represented as a matrix group, we used the MAGMA commands "O:= Orbits(G)", "#O", "#O[1]", "#O[2]",#O[3]", and "#O[4]" to compute the orbit lengths of the action of G on N. We obtain 4 orbits of lengths 1, 21, 210, and 280 and using the MAGMA commands "P1:= Stabilizer(G,O[1, 1])", "P2:= Stabilizer(G,O[2, 1])", "P3:= Stabilizer(G,O[3, 1])", and "P4:= Stabilizer(G,O[4, 1])", we are able to compute the corresponding point stabilizers P_i, $i = 1, 2, 3, 4$, which are subgroups of G. With the aid of MAGMA and also checking the indices of the maximal subgroups of G in the ATLAS, the structures of the stabilizers P_i are identified as $P_1 = L_3(4):2$, $P_2 = 2^4 : S_5$, $P_3 = 2^4 : (2 \times S_3)$, and $P_4 = 3^2 : (Q_8.2)$, where P_2 and P_4 are maximal subgroups of P_1. We should note here that the group $L_3(4) : 2$ has two nonconjugate isomorphic maximal subgroups $L_1 = P_2$ and L_2, having the same structure $2^4 : S_5$. The stabilizer P_3 sits maximally in L_2. Alternatively, we can use [13] to identify the structures of the groups P_i. Since the action of G on N does not fix any nontrivial subspace of 2^9, we have that 2^9 is an irreducible module for G. We can readily verify this fact by using the MAGMA command "IsIrreducible(G)".

Let $\chi(L_3(4) : 2 \mid 2^9)$ be the permutation character of $L_3(4):2$ on the classes of 2^9. Then, from methods that were developed by Mpono [14], we obtain that $\chi(L_3(4):2 \mid 2^9) = I_{P_1}^{P_1} + I_{P_2}^{P_1} + I_{P_3}^{P_1} + I_{P_4}^{P_1} = 4 \times 1a + 4 \times 20a + 2 \times 35a + 45a + 45b + 2 \times 64a + 2 \times 70a$, where $I_{P_1}^{P_1}$, $I_{P_2}^{P_1}$, $I_{P_3}^{P_1}$, and $I_{P_4}^{P_1}$ are the identity characters of the point stabilizers P_i, $i = 1, 2, 3, 4$, induced to G. Note that the identity characters $I_{P_i}^{P_1}$ are identified with the permutation characters $\chi(L_3(4) : 2 \mid P_i)$ of $L_3(4):2$ acting on the classes of the point stabilizers P_i. We found that $I_{P_1}^{P_1} = \chi(L_3(4):3 \mid P_1) = 1a$, $I_{P_2}^{P_1} = \chi(L_3(4):2 \mid P_2) = 1a + 20a$, $I_{P_3}^{P_1} = \chi(L_3(4):2 \mid P_3) = 1a + 2 \times 20a + 35a + 64a + 70a$, and $I_{P_4}^{P_1} = \chi(L_3(4):3 \mid P_4) = 1a + 20a + 35a + 45a + 45b + 64a + 70a$. The permutation characters $\chi(L_3(4) : 2 \mid P_i)$ are written in terms of the ordinary irreducible characters of G. Since we have the generators g_1 and g_2 for G, we compute the character tables of G and the P_i's directly in MAGMA and use these tables together with the fusion maps of the stabilizers into G, to compute $\chi(L_3(4):2 \mid P_i)$ and $\chi(L_3(4):2 \mid 2^9)$. The values of $\chi(L_3(4):2 \mid 2^9)$ on the different classes of G determine the number k of fixed points of each $g \in G$ in 2^9. The values of k are listed in Table 1.

The values of k enabled us to determine the number f_j of orbits Q_i's, $1 \leq i \leq k$, which have fused together under the action of $C_G(g)$, for each class representative $g \in G$, to form one orbit $\triangle_f$. Mpono in [14] used the technique of coset analysis to develop Programmes A and B in CAYLEY [15] for the computation of the conjucacy classes of a split extension $\overline{G} = N : G$, where N is an elementary abelian p-group for a prime p on which a linear group G acts. Ali [16] adapted Programmes A and B to be used in MAGMA. Programme A computes the values of the $f_j's$, whereas Programme B determines the order of the elements for each conjugacy class $[x]$ in $\overline{G}$. We obtain that $\overline{G}$ has exactly 49 conjugacy classes. The parameters d_j and w are defined in [14]. The centralizer order of each class $[x]$ of $\overline{G}$ is computed using the formula $|C_{\overline{G}}(x)| = (k/f)|C_G(g)|$. All the information involving the conjugacy classes of $\overline{G}$ are listed in Table 2.

4. The Inertia Groups of $\overline{G}$=2^9:($L_3(4)$:2)

Since G has four orbits on N, then by Brauer's Theorem [17]G acts on $Irr(N)$ with the same number of orbits. The lengths of the 4 orbits will be 1,r, s, and t where $r + s + t = 511$, with corresponding point stabilizers H_1, H_2, H_3, and H_4 as subgroups of G such that $[G : H_1]$ =1, $[G : H_2]$ =r, $[G : H_3]$= s, and $[G : H_4] = t$. We generate G as a permutation group on a set of cardinality 672 within MAGMA. Then the maximal and submaximal subgroups of G are computed. Now, considering the indices of these subgroups in G, the number of the classes of these subgroups, and also the fact that $\overline{G}$ has 49 conjugacy classes, we deduce that the action of G on $Irr(N)$ has orbits of lengths 1, $r = 21$, $s = 210$, and $t = 280$ with respective point stabilizers $H_1 = L_3(4):2$, $H_2 = 2^4 : S_5$, $H_3 = 2^4 : (2 \times S_3)$, and $H_4 = 3^2 : (Q_8 : 2)$. Thus we obtain four inertia groups $\overline{H}_i = 2^9 : H_i$, $i \in \{1, 2, 3, 4\}$, for $2^9 : (L_3(4) : 2)$ on Irr(N). Alternatively, we can also determine the inertia factor groups if we let T be the matrix group of dimension 9 over $GF(2)$ formed by the transpose of the generators of G. Then the action of T on the classes of $N = 2^9$ is the equivalent of G acting on $Irr(N)$. Then with the help of MAGMA or GAP, we can easily verify that the action of T on N has orbits of lengths 1, 21, 210, and 280 with corresponding point stabilizers T, $2^4 : S_5$, $2^4 : (2 \times S_3)$, and $3^2 : (Q_8 : 2)$. The structures of H_2 and H_4 have been identified by checking the indices of the maximal subgroups of $L_3(4) : 2 \cong L_3(4).2_2$ in the ATLAS. The structure of H_3 was determined by direct computations

TABLE 2: The conjugacy classes of elements of $\overline{G} = 2^9 : (L_3(4) : 2)$.

$[g]_G$	k	f_j	d_j	w	$[x]_{\overline{G}}$	$\lvert[x]_{\overline{G}}\rvert$	$\lvert C_{\overline{G}}(x)\rvert$
$1A$	512	$f_1 = 1$	$(0,0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$1A$	1	20643840
		$f_2 = 21$	$(0,0,0,0,1,1,0,1,1)$	$(0,0,0,0,1,1,0,1,1)$	$2A$	21	983040
		$f_3 = 210$	$(1,0,0,0,0,0,0,0,0)$	$(1,0,0,0,0,0,0,0,0)$	$2B$	210	98304
		$f_4 = 280$	$(1,0,1,0,1,1,1,0,1)$	$(1,0,1,0,1,1,1,0,1)$	$2C$	280	73728
$2A$	64	$f_1 = 1$	$(0,0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$2D$	960	21504
		$f_2 = 7$	$(0,0,0,0,0,1,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$2E$	6720	3072
		$f_3 = 7$	$(0,0,0,0,0,0,0,1,1)$	$(0,1,1,1,1,0,0,1,0)$	$4A$	6720	3072
		$f_4 = 21$	$(1,0,0,0,1,1,1,0,0)$	$(0,0,0,0,1,1,1,0,0)$	$4B$	20160	1024
		$f_5 = 28$	$(0,0,0,0,0,0,0,0,1)$	$(0,1,1,1,0,1,1,1,0)$	$4C$	26880	768
$2B$	32	$f_1 = 1$	$(0,0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$2F$	5040	4096
		$f_2 = 1$	$(0,0,0,0,0,0,0,1,1)$	$(0,0,0,0,0,0,0,0,0)$	$2G$	5040	4096
		$f_3 = 1$	$(1,1,1,1,1,1,1,1,1)$	$(0,1,1,1,1,0,0,0,1)$	$4D$	5040	4096
		$f_4 = 1$	$(1,1,0,0,0,0,0,1,1)$	$(0,1,1,1,1,0,0,0,1)$	$4E$	5040	4096
		$f_5 = 4$	$(1,1,1,0,1,0,0,1,1)$	$(0,0,1,0,0,1,1,0,0)$	$4F$	20160	1024
		$f_6 = 8$	$(0,1,1,1,0,1,0,1,0)$	$(0,0,1,1,1,1,1,1,1)$	$4G$	40320	512
		$f_7 = 8$	$(1,1,1,1,1,0,1,1,1)$	$(0,0,0,0,1,1,1,1,0)$	$4H$	40320	512
		$f_8 = 8$	$(1,0,0,0,0,0,0,0,0)$	$(0,0,1,1,0,0,0,0,1)$	$4I$	40320	512
$3A$	8	$f_1 = 1$	$(0,0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$3A$	143360	144
		$f_2 = 1$	$(1,0,0,1,0,1,1,1,1)$	$(1,0,0,1,0,1,1,1,1)$	$6A$	143360	144
		$f_3 = 3$	$(1,1,1,1,0,1,0,0,1)$	$(1,1,1,1,0,1,0,0,1)$	$6B$	430080	48
		$f_4 = 3$	$(0,0,0,0,0,0,0,1,0)$	$(1,0,0,1,1,0,1,1,0)$	$6C$	430080	48
$4A$	8	$f_1 = 1$	$(0,0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$4J$	80640	256
		$f_2 = 1$	$(0,1,0,0,1,0,0,1,1)$	$(0,0,0,0,0,0,0,0,0)$	$4K$	80640	256
		$f_3 = 2$	$(0,0,1,0,0,0,0,0,0)$	$(0,1,1,1,1,0,0,0,1)$	$8A$	161280	128
		$f_4 = 4$	$(1,0,1,0,1,0,1,1,1)$	$(0,1,0,1,1,1,1,0,1)$	$8B$	322560	64
$4B$	8	$f_1 = 1$	$(0,0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$4L$	161280	128
		$f_2 = 1$	$(1,0,0,0,0,0,0,1,0)$	$(0,0,0,0,0,0,0,0,0)$	$4M$	161280	128
		$f_3 = 2$	$(1,0,0,0,1,1,1,1,1)$	$(0,0,1,1,0,0,0,1,0)$	$8C$	322560	64
		$f_4 = 4$	$(1,0,0,0,0,0,0,0,0)$	$(1,1,0,1,1,0,0,1,0)$	$8D$	645120	32
$4C$	8	$f_1 = 1$	$(0,0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$4N$	161280	128
		$f_2 = 1$	$(1,0,1,1,1,0,1,1,1)$	$(0,0,0,0,0,0,0,0,0)$	$4O$	161280	128
		$f_3 = 1$	$(1,0,0,0,1,1,0,1,0)$	$(1,0,1,0,1,1,1,1,0)$	$8E$	161280	128
		$f_4 = 1$	$(0,1,1,1,1,1,1,1,0)$	$(1,0,1,0,1,1,1,1,0)$	$8F$	161280	128
		$f_5 = 2$	$(1,0,0,0,0,0,0,0,0)$	$(1,1,0,1,1,1,0,0,1)$	$8G$	322560	64
		$f_6 = 2$	$(1,0,1,0,0,1,0,0,1)$	$(0,1,1,1,0,0,1,1,1)$	$8H$	322560	64
$5A$	2	$f_1 = 1$	$(0,0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$5A$	2064384	10
		$f_2 = 1$	$(1,0,0,0,0,0,0,0,0)$	$(0,0,0,0,1,1,0,0,1)$	$10A$	2064384	10
$6A$	4	$f_1 = 1$	$(0,0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$6D$	860160	24
		$f_2 = 1$	$(1,0,0,0,1,0,1,1,0)$	$(0,0,0,0,0,0,0,0,0)$	$6E$	860160	24
		$f_3 = 1$	$(1,0,0,1,1,0,1,1,1)$	$(1,0,0,1,0,1,1,1,0)$	$12A$	860160	24
		$f_4 = 1$	$(1,1,1,0,1,0,1,1,1)$	$(1,0,0,1,0,1,1,1,0)$	$12B$	860160	24
$7A$	1	$f_1 = 1$	$(0,0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$7A$	1474560	14
$7B$	1	$f_1 = 1$	$(0,0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$7B$	1474560	14
$8A$	4	$f_1 = 1$	$(0,0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$8I$	645120	32
		$f_2 = 1$	$(1,0,0,1,1,1,1,1,1)$	$(0,0,0,0,0,0,0,0,0)$	$8J$	645120	32
		$f_3 = 1$	$(1,1,1,0,1,0,1,1,1)$	$(0,0,0,0,0,0,0,1,1)$	$16A$	645120	32
		$f_4 = 1$	$(1,1,1,1,1,1,0,0,0)$	$(0,0,0,0,0,0,0,1,1)$	$16B$	645120	32
$14A$	1	$f_1 = 1$	$(0,0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$14A$	1474560	14
$14B$	1	$f_1 = 1$	$(0,0,0,0,0,0,0,0,0)$	$(0,0,0,0,0,0,0,0,0)$	$14B$	1474560	14

in MAGMA. The groups H_2, H_3, and H_4 are constructed from elements within G and the generators are as follows:

(i) $H_2 = \langle \alpha_1, \alpha_2 \rangle$, $\alpha_1 \in 3A$, and $\alpha_2 \in 6A$ where

$$\alpha_1 = \begin{pmatrix} 0&0&0&0&1&0&1&0&0\\ 0&1&0&0&1&0&1&0&0\\ 1&0&0&1&0&0&1&1&1\\ 1&1&1&0&1&1&1&0&0\\ 0&0&0&0&0&0&1&1&0\\ 0&0&0&0&0&1&0&0&0\\ 1&0&0&0&1&0&0&1&0\\ 1&0&0&0&0&1&1&0&0\\ 0&0&1&1&0&1&0&1&0 \end{pmatrix}, \quad \alpha_2 = \begin{pmatrix} 1&0&0&0&1&1&0&0&0\\ 0&1&1&1&1&0&0&1&0\\ 1&1&0&0&1&0&1&1&1\\ 1&0&1&0&0&0&0&1&0\\ 1&0&0&0&0&1&1&0&0\\ 0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&1&0\\ 0&0&0&0&0&1&1&1&0\\ 0&0&1&1&0&1&1&0&0 \end{pmatrix} \tag{3}$$

(ii) $H_3 = \langle \beta_1, \beta_2 \rangle$, $\beta_1 \in 4C$, and $\beta_2 \in 6A$ where

$$\beta_1 = \begin{pmatrix} 1&0&0&0&0&0&0&0&0\\ 1&1&0&1&0&1&1&0&0\\ 0&0&0&1&1&1&1&1&1\\ 0&0&1&1&1&1&0&1&1\\ 1&0&0&1&1&1&0&0&0\\ 1&0&1&0&0&0&1&1&1\\ 1&0&0&1&0&1&1&0&0\\ 1&1&0&1&0&1&1&0&1\\ 1&1&0&1&1&0&1&1&0 \end{pmatrix}, \quad \beta_2 = \begin{pmatrix} 1&0&0&0&0&0&0&0&0\\ 1&0&0&1&1&1&0&0&1\\ 1&0&0&1&1&1&1&1&1\\ 0&1&0&0&1&0&0&0&1\\ 1&0&0&1&1&1&0&0&0\\ 1&0&1&0&0&0&1&1&1\\ 0&0&1&1&0&1&0&1&1\\ 1&1&0&1&0&1&1&0&1\\ 1&1&0&1&1&0&1&1&0 \end{pmatrix} \tag{4}$$

(iii) $H_4 = \langle \gamma_1, \gamma_2 \rangle$, $\gamma_1 \in 2A$, adn $\gamma_2 \in 8A$ where

$$\gamma_1 = \begin{pmatrix} 0&0&0&0&1&0&1&0&0\\ 0&1&0&0&0&0&0&0&0\\ 1&0&0&1&1&0&0&1&1\\ 1&0&0&1&1&0&1&0&0\\ 1&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&1&1&1&0\\ 0&0&1&1&0&1&0&1&0 \end{pmatrix}, \quad \gamma_2 = \begin{pmatrix} 1&1&0&1&0&0&0&0&0\\ 1&1&0&0&0&1&1&1&1\\ 0&1&1&1&1&1&1&0&0\\ 0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&1&1\\ 0&0&0&0&1&1&0&0&1\\ 0&0&0&1&1&0&0&1&1\\ 0&0&0&0&1&1&0&0&0\\ 0&0&0&0&1&0&0&0&1 \end{pmatrix} \tag{5}$$

For the purpose of constructing the character table of $\overline{G}$, we use the above generators of the H_i's to compute their character tables.

5. The Fusion of H_2, H_3, and H_4 into G

We obtain the fusions of the inertia factors H_2, H_3, and H_4 into G by using direct matrix conjugation in G and their permutation characters in G of degrees 21, 210, and 280, respectively. MAGMA was used for the various computations. The fusion maps of H_2, H_3, and H_4 into G are shown in Tables 3, 4, and 5.

6. The Fischer-Clifford Matrices of $2^9{:}(L_3(4){:}2)$

Having obtained the fusions of the inertia factors into $L_3(4)$: 2 and the conjugacy classes of $L_3(4)$: 2 displayed in the format of Table 2, we can proceed to use the theory and properties discussed in [9] or [14] to help us in the construction of the Fischer-Clifford matrices of $2^9 : (L_3(4) : 2)$. Note that all the relations hold since 2^9 is an elementary abelian group.

For example, consider the conjugacy class $2A$ of $L_3(4)$: 2. Then we obtain that $M(2A)$ has the following form with corresponding weights attached to the rows and columns:

TABLE 3: The fusion of H_2 into $L_3(4):2$.

$[h]_{H_2}$	$\longrightarrow$	$[g]_{L_3(4):2}$	$[h]_{H_2}$	$\longrightarrow$	$[g]_{L_3(4):2}$	$[h]_{H_2}$	$\longrightarrow$	$[g]_{L_3(4):2}$	$[h]_{H_2}$	$\longrightarrow$	$[g]_{L_3(4):2}$
$1A$		$1A$	$2C$		$2B$	$4B$		$4B$	$5A$		$5A$
$2A$		$2B$	$3A$		$3A$	$4C$		$4C$	$6A$		$6A$
$2B$		$2A$	$4A$		$4A$	$4D$		$4C$	$8A$		$8A$

TABLE 4: The fusion of H_3 into $L_3(4):2$.

$[h]_{H_3}$	$\longrightarrow$	$[g]_{L_3(4):2}$	$[h]_{H_3}$	$\longrightarrow$	$[g]_{L_3(4):2}$	$[h]_{H_3}$	$\longrightarrow$	$[g]_{L_3(4):2}$	$[h]_{H_3}$	$\longrightarrow$	$[g]_{L_3(4):2}$
$1A$		$1A$	$2D$		$2B$	$4A$		$4C$	$4E$		$4B$
$2A$		$2B$	$2E$		$2B$	$4B$		$4C$	$6A$		$6A$
$2B$		$2A$	$2F$		$2A$	$4C$		$4A$			
$2C$		$2B$	$3A$		$3A$	$4D$		$4C$			

$$
\begin{aligned}
M(2A) &= \begin{array}{c} \\ |C_G(2A)| \\ |C_{H_2}(2B)| \\ |C_{H_3}(2B)| \\ |C_{H_3}(2F)| \\ |C_{H_4}(2B)| \\ \\ \end{array}
\begin{array}{c} \begin{array}{ccccc} |C_{\overline{G}}(2D)| & |C_{\overline{G}}(2E)| & |C_{\overline{G}}(4A)| & |C_{\overline{G}}(4B)| & |C_{\overline{G}}(4C)| \end{array} \\
\begin{pmatrix} a & f & k & p & u \\ b & g & l & q & v \\ c & h & m & r & w \\ d & i & n & s & x \\ e & j & o & t & y \end{pmatrix} \\
\begin{array}{ccccc} m_1 & m_2 & m_3 & m_4 & m_5 \end{array} \end{array} \\
&= \begin{array}{c} \\ 336 \\ 48 \\ 48 \\ 16 \\ 12 \\ \\ \end{array}
\begin{array}{c} \begin{array}{ccccc} 21504 & 3072 & 3072 & 1024 & 768 \end{array} \\
\begin{pmatrix} a & f & k & p & u \\ b & g & l & q & v \\ c & h & m & r & w \\ d & i & n & s & x \\ e & j & o & t & y \end{pmatrix} \\
\begin{array}{ccccc} 8 & 56 & 56 & 168 & 224 \end{array} \end{array}.
\end{aligned} \tag{6}
$$

We have $a = f = k = p = u = 1, b = c = 7, d = 21$, and $e = 28$ by using Theorem 5.2.4 and property (e) of the Fischer-Clifford matrix $M(g)$ (both found in [14]). Thus we obtain the following form:

$$
M(2A) = \begin{array}{c} \\ 336 \\ 48 \\ 48 \\ 16 \\ 12 \\ \\ \end{array}
\begin{array}{c} \begin{array}{ccccc} 21504 & 3072 & 3072 & 1024 & 768 \end{array} \\
\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 7 & g & l & q & v \\ 7 & h & m & r & w \\ 21 & i & n & s & x \\ 28 & j & o & t & y \end{pmatrix} \\
\begin{array}{ccccc} 8 & 56 & 56 & 168 & 224 \end{array} \end{array}. \tag{7}
$$

By the orthogonality relations for columns and rows and remaining properties of the matrix $M(g)$ found in Chapter 5 of [14], we obtain the desired Fischer-Clifford matrix $M(2A)$ of $\overline{G}$ given below:

$$
M(2A) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 7 & -1 & -5 & 3 & -1 \\ 7 & 7 & -1 & -1 & -1 \\ 21 & -3 & 9 & 1 & -3 \\ 28 & -4 & -4 & -4 & 4 \end{pmatrix}. \tag{8}
$$

For each class representative $g \in L_3(4):2$, we construct a Fischer-Clifford matrix $M(g)$. These are listed in Table 6.

7. Character Table of $2^9{:}(L_3(4){:}2)$

Having obtained the Fischer-Clifford matrices, the fusion maps of the H_i's into $L_3(4) : 2$, and the character tables of the inertia factors H_i, we construct the character table of $2^9 : (L_3(4) : 2)$ following the methodology discussed in Section 5.2 of [14]. For example, we calculate the partial character table of $2^9 : (L_3(4) : 2)$ corresponding to the coset of $2A \in L_3(4) : 2$. From the Fischer-Clifford matrix $M(2A)$ we obtain that

$$M_1(2A) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \end{pmatrix},$$
$$M_2(2A) = \begin{pmatrix} 7 & -1 & -5 & 3 & -1 \end{pmatrix},$$
$$M_3(2A) = \begin{pmatrix} 7 & 7 & -1 & -1 & -1 \\ 21 & -3 & 9 & 1 & -3 \end{pmatrix} \tag{9}$$
$$\text{and } M_4(2A) = \begin{pmatrix} 28 & -4 & -4 & -4 & 4 \end{pmatrix}.$$

Let $C_1(2A)$, $C_2(2A)$, $C_3(2A)$, and $C_4(2A)$ be the partial character tables of the inertia factors for the classes which fuse to $2A \in L_3(4) : 2$. Then the partial character table of $2^9 : (L_3(4) : 2)$ on the classes $\{2D, 2E, 4A, 4B, 4C\}$ is given by

$$C_1(2A)\,M_1(2A) = \begin{pmatrix} 1 \\ -1 \\ -6 \\ 6 \\ -7 \\ 7 \\ -3 \\ -3 \\ 3 \\ 3 \\ 8 \\ -8 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \end{pmatrix}
= \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 \\ -6 & -6 & -6 & -6 & -6 \\ 6 & 6 & 6 & 6 & 6 \\ -7 & -7 & -7 & -7 & -7 \\ 7 & 7 & 7 & 7 & 7 \\ -3 & -3 & -3 & -3 & -3 \\ -3 & -3 & -3 & -3 & -3 \\ 3 & 3 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 & 3 \\ 8 & 8 & 8 & 8 & 8 \\ -8 & -8 & -8 & -8 & -8 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$C_2(2A)\,M_2(2A) = \begin{pmatrix} 1 \\ -1 \\ 2 \\ -2 \\ -1 \\ 1 \\ 0 \\ -3 \\ -3 \\ 3 \\ 3 \\ 0 \end{pmatrix} \begin{pmatrix} 7 & -1 & -5 & 3 & -1 \end{pmatrix}
= \begin{pmatrix} 7 & -1 & -5 & 3 & -1 \\ -7 & 1 & 5 & -3 & 1 \\ 14 & -2 & -10 & 6 & -2 \\ -14 & 2 & 10 & -6 & 2 \\ -7 & 1 & 5 & -3 & 1 \\ 7 & -1 & -5 & 3 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ -21 & 3 & 15 & -9 & 3 \\ -21 & 3 & 15 & -9 & 3 \\ 21 & -3 & -15 & 9 & -3 \\ 21 & -3 & -15 & 9 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$C_3(2A)\,M_3(2A) = \begin{pmatrix} 1 & 1 \\ -1 & -1 \\ 1 & -1 \\ -1 & 1 \\ -2 & 0 \\ 2 & 0 \\ 3 & -1 \\ -3 & -1 \\ 3 & 1 \\ -3 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & -2 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 7 & 7 & -1 & -1 & -1 \\ 21 & -3 & 9 & 1 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} 28 & 4 & 8 & 0 & -4 \\ -28 & -4 & -8 & 0 & 4 \\ -14 & 10 & -10 & -2 & 2 \\ 14 & -10 & 10 & 2 & -2 \\ -14 & -14 & 2 & 2 & 2 \\ 14 & 14 & -2 & -2 & -2 \\ 0 & 24 & -12 & -4 & 0 \\ -42 & -18 & -6 & 2 & 6 \\ 42 & 18 & 6 & -2 & -6 \\ 0 & -24 & 12 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -42 & 6 & -18 & -2 & 6 \\ 42 & -6 & 18 & 2 & -6 \end{pmatrix}$$

$$C_4(2A)\,M_4(2A) = \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \\ 0 \\ 0 \\ 0 \\ -2 \\ 2 \end{pmatrix} \begin{pmatrix} 28 & -4 & -4 & -4 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 28 & -4 & -4 & -4 & 4 \\ -28 & 4 & 4 & 4 & -4 \\ -28 & 4 & 4 & 4 & -4 \\ 28 & -4 & -4 & -4 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -56 & 8 & 8 & 8 & -8 \\ 56 & -8 & -8 & -8 & 8 \end{pmatrix} \quad (10)$$

Similarly, the partial character table associated with each coset Ng is computed. If necessary, we will restrict some characters of $Irr(U_6(2):2)$ to $\overline{G}$, to ensure that each partial character table corresponding to a coset Ng will give rise to the desired set $Irr(\overline{G})$.

The character table of $\overline{G}$ will be partitioned row-wise into 4 blocks Δ_1, Δ_2, Δ_3, and Δ_4 where each block corresponds to an inertia group $\overline{H}_i = 2^9 : H_i$. Therefore $Irr(2^9 : (L_3(4) : 2)) = \bigcup_{i=1}^{4} \Delta_i$, where $\Delta_1 = \{\chi_j \mid 1 \le j \le 14\}$, $\Delta_2 = \{\chi_j \mid 15 \le j \le 26\}$, $\Delta_3 = \{\chi_j \mid 27 \le j \le 40\}$, and $\Delta_4 = \{\chi_j \mid 41 \le j \le 49\}$. The character table of $2^9 : (L_3(4) : 2)$ is shown in Table 7. The consistency and accuracy of the character table of $2^9 : (L_3(4) : 2)$ have been tested by using the GAP code labelled as Programme E in [18].

Table 5: The fusion of H_4 into $L_3(4):2$.

$[h]_{H_4}$	$\longrightarrow$ $[g]_{L_3(4):2}$	$[h]_{H_4}$	$\longrightarrow$ $[g]_{L_3(4):2}$	$[h]_{H_4}$	$\longrightarrow$ $[g]_{L_3(4):2}$
$1A$	$1A$	$3A$	$3A$	$6A$	$6A$
$2A$	$2B$	$4A$	$4A$	$8A$	$8A$
$2B$	$2A$	$4B$	$4B$	$8B$	$8A$

The information about the conjugacy classes found in Table 2 can be used to compute the power maps for the elements of $\overline{G}$ and then with the aid of Programme E in [18] we can verify that we obtained the unique p-power maps listed in Table 8 for our Table 7.

8. The Fusion of 2^9:($L_3(4)$:2) into $U_6(2)$:2

Since $\overline{G}$ is a maximal subgroup of $U_6(2):2$ of index 891, then the action of $U_6(2):2$ on the cosets of $\overline{G}$ gives rise to a permutation character $\chi(U_6(2):2 \mid \overline{G})$ of degree 891. We deduce from the character table of $U_6(2):2$ found in GAP that $\chi(U_6(2):2 \mid \overline{G}) = 1a + 22a + 252a + 616a$, where $1a$, $22a$, $252a$, and $616a$ are irreducible characters of $U_6(2):2$ of degrees 1, 22, 252, and 616, respectively.

We are able to obtain the partial fusion of $\overline{G}$ into $U_6(2):2$, using the information provided by the values of $\chi(U_6(2):2 \mid \overline{G})$ on the classes of $\overline{G}$ and the power maps of $\overline{G}$ and $U_6(2):2$. Then, the technique of set intersections for characters (see [9, 14, 19]) is applied to restrict some ordinary irreducible characters of $U_6(2):2$ of small degrees to $\overline{G}$, to determine fully the fusion of the classes of $\overline{G}$ into $U_6(2):2$.

Let ζ be the character afforded by the regular representation of $L_3(4):2$. We obtain that $\zeta = \Sigma_{i=1}^{14}\alpha_i\Phi_i$, where $\Phi_i \in Irr(L_3(4):2)$ and $\alpha_i = \deg(\Phi_i)$. Then ζ can be regarded as a character of $2^9 : (L_3(4):2)$ which contains 2^9 in its kernel such that

$$\zeta(x) = \begin{cases} |L_3(4):2| & \text{if } x \in 2^9 \\ 0 & \text{otherwise.} \end{cases} \quad (11)$$

If ϕ is a character of $U_6(2):2$ than we have that

$$\begin{aligned} \langle \zeta, \phi \rangle_{\overline{G}} &= \frac{1}{|2^9 : (L_3(4):2)|}\{\zeta(1A)\phi(1A) \\ &\quad + 21\zeta(2A)\phi(2A) + 210\zeta(2B)\phi(2B) + 280\zeta(2C) \\ &\quad \cdot \phi(2C)\} = \frac{1}{|2^9 : (L_3(4):2|}\{|L_3(4):2| \\ &\quad \cdot (\phi(1A) + 21\phi(2A) + 210\phi(2B) + 280\phi(2C))\} \\ &= \frac{1}{512}\{\phi(1A) + 21\phi(2A) + 210\phi(2B) \\ &\quad + 280\phi(2C)\} = \langle \phi_{2^9}, 1_{2^9} \rangle. \end{aligned} \quad (12)$$

Table 6: The Fischer-Clifford matrices of $2^9 : (L_3(4) : 2)$.

$M(g)$	$M(g)$
$M(1A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 21 & -11 & 5 & -3 \\ 210 & 50 & 2 & -6 \\ 280 & -40 & -8 & 8 \end{pmatrix}$	$M(2A) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 7 & -1 & -5 & 3 & -1 \\ 7 & 7 & -1 & -1 & -1 \\ 21 & -3 & 9 & 1 & -3 \\ 28 & -4 & -4 & -4 & 4 \end{pmatrix}$
$M(2B) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 \\ 4 & -4 & 4 & -4 & 0 & -2 & 2 & 0 \\ 2 & 2 & 2 & 2 & 2 & 0 & 0 & -2 \\ 4 & -4 & 4 & -4 & 0 & 2 & -2 & 0 \\ 4 & 4 & 4 & 4 & -4 & 0 & 0 & 0 \\ 8 & 8 & -8 & -8 & 0 & 0 & 0 & 0 \\ 8 & -8 & -8 & 8 & 0 & 0 & 0 & 0 \end{pmatrix}$	$M(3A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 3 & -3 & -1 & 1 \\ 3 & 3 & -1 & -1 \\ 1 & -1 & 1 & -1 \end{pmatrix}$
$M(4A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 2 & 2 & -2 & 0 \\ 4 & -4 & 0 & 0 \end{pmatrix}$	$M(4B) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 2 & 2 & -2 & 0 \\ 4 & -4 & 0 & 0 \end{pmatrix}$
$M(4C) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 & -1 & 1 \\ 2 & -2 & 2 & -2 & 0 & 0 \\ 1 & 1 & 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 1 & -1 \\ 2 & -2 & -2 & 2 & 0 & 0 \end{pmatrix}$	$M(5A) = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$
$M(6A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \end{pmatrix}$	$M(7A) = (1)$
$M(7B) = (1)$	$M(8A) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{pmatrix}$
$M(14A) = (1)$	$M(14B) = (1)$

Here 1_{2^9} is the identity character of 2^9 and ϕ_{2^9} is the restriction of ϕ to 2^9. We obtain that

$$\phi_{2^9} = a_1\theta_1 + a_2\theta_2 + a_3\theta_3 + a_4\theta_4, \tag{13}$$

where $a_i \in \mathbb{N} \cup \{0\}$ and θ_i are the sums of the irreducible characters of 2^9 which are in the same orbit under the action of $L_3(4) : 2$ on $Irr(2^9)$, for $i \in \{1, 2, 3, 4\}$. Let $\varphi_j \in Irr(2^9)$, where $j \in \{1, 2, 3, \ldots, 512\}$. Then we obtain that

$$\theta_1 = \varphi_1, \quad \deg(\theta_1) = 1$$

$$\theta_2 = \sum_{j=2}^{22} \varphi_j, \quad \deg(\theta_2) = 21$$

TABLE 7: The character table of $2^9 : (L_3(4) : 2)$.

	1A				2A					2B		
	$1A$	$2A$	$2B$	$2C$	$2D$	$2E$	$4A$	$4B$	$4C$	$2F$	$2G$	$4D$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	-1	-1	-1	-1	-1	1	1	1
χ_3	20	20	20	20	-6	-6	-6	-6	-6	4	4	4
χ_4	20	20	20	20	6	6	6	6	6	4	4	4
χ_5	35	35	35	35	-7	-7	-7	-7	-7	3	3	3
χ_6	35	35	35	35	7	7	7	7	7	3	3	3
χ_7	45	45	45	45	-3	-3	-3	-3	-3	-3	-3	-3
χ_8	45	45	45	45	-3	-3	-3	-3	-3	-3	-3	-3
χ_9	45	45	45	45	3	3	3	3	3	-3	-3	-3
χ_{10}	45	45	45	45	3	3	3	3	3	-3	-3	-3
χ_{11}	64	64	64	64	8	8	8	8	8	0	0	0
χ_{12}	64	64	64	64	-8	-8	-8	-8	-8	0	0	0
χ_{13}	70	70	70	70	0	0	0	0	0	6	6	6
χ_{14}	126	126	126	126	0	0	0	0	0	-2	-2	-2
χ_{15}	21	-11	5	-3	7	-1	-5	3	-1	5	-3	5
χ_{16}	21	-11	5	-3	-7	1	5	-3	1	5	-3	5
χ_{17}	84	-44	20	-12	14	-2	-10	6	-2	4	4	4
χ_{18}	84	-44	20	-12	-14	2	10	-6	2	4	4	4
χ_{19}	105	-55	25	-15	-7	1	5	-3	1	9	1	9
χ_{20}	105	-55	25	-15	7	-1	-5	3	-1	9	1	9
χ_{21}	126	-66	30	-18	0	0	0	0	0	-2	14	-2
χ_{22}	315	-165	75	-45	-21	3	15	-9	3	11	-13	11
χ_{23}	315	-165	75	-45	-21	3	15	-9	3	-5	3	-5
χ_{24}	315	-165	75	-45	21	-3	-15	9	-3	11	-13	11
χ_{25}	315	-165	75	-45	21	-3	-15	9	-3	-5	3	-5
χ_{26}	630	-330	150	-90	0	0	0	0	0	-10	6	-10
χ_{27}	210	50	2	-6	28	4	8	0	-4	18	10	2
χ_{28}	210	50	2	-6	-28	-4	-8	0	4	18	10	2
χ_{29}	210	50	2	-6	-14	10	-10	-2	2	2	-6	18
χ_{30}	210	50	2	-6	14	-10	10	2	-2	2	-6	18
χ_{31}	420	100	4	-12	-14	-14	2	2	2	20	4	20
χ_{32}	420	100	4	-12	14	14	-2	-2	-2	20	4	20
χ_{33}	630	150	6	-18	0	24	-12	-4	0	-10	-2	6
χ_{34}	630	150	6	-18	-42	-18	-6	2	6	6	14	-10
χ_{35}	630	150	6	-18	42	18	6	-2	-6	6	14	-10
χ_{36}	630	150	6	-18	0	-24	12	4	0	-10	-2	6
χ_{37}	1260	300	12	-36	0	0	0	0	0	-20	-4	12
χ_{38}	1260	300	12	-36	0	0	0	0	0	12	28	-20
χ_{39}	1260	300	12	-36	-42	6	-18	-2	6	-4	-20	-4
χ_{40}	1260	300	12	-36	42	-6	18	2	-6	-4	-20	-4
χ_{41}	280	-40	-8	8	28	-4	-4	-4	4	8	-8	-8
χ_{42}	280	-40	-8	8	-28	4	4	4	-4	8	-8	-8
χ_{43}	280	-40	-8	8	-28	4	4	4	-4	8	-8	-8
χ_{44}	280	-40	-8	8	28	-4	-4	-4	4	8	-8	-8
χ_{45}	560	-80	-16	16	0	0	0	0	0	16	-16	-16
χ_{46}	560	-80	-16	16	0	0	0	0	0	-16	16	16

Table 7: Continued.

χ_{47}	560	-80	-16	16	0	0	0	0	0	-16	16	16	
χ_{48}	2240	-320	-64	64	-56	8	8	8	-8	0	0	0	
χ_{49}	2240	-320	-64	64	56	-8	-8	-8	8	0	0	0	
			$2B$					$3A$				$4A$	
	$4E$	$4F$	$4G$	$4H$	$4I$	$3A$	$6A$	$6B$	$6C$	$4J$	$4K$	$8A$	$8B$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_3	4	4	4	4	4	2	2	2	2	0	0	0	0
χ_4	4	4	4	4	4	2	2	2	2	0	0	0	0
χ_5	3	3	3	3	3	-1	-1	-1	-1	3	3	3	3
χ_6	3	3	3	3	3	-1	-1	-1	-1	3	3	3	3
χ_7	-3	-3	-3	-3	-3	0	0	0	0	1	1	1	1
χ_8	-3	-3	-3	-3	-3	0	0	0	0	1	1	1	1
χ_9	-3	-3	-3	-3	-3	0	0	0	0	1	1	1	1
χ_{10}	-3	-3	-3	-3	-3	0	0	0	0	1	1	1	1
χ_{11}	0	0	0	0	0	1	1	1	1	0	0	0	0
χ_{12}	0	0	0	0	0	1	1	1	1	0	0	0	0
χ_{13}	6	6	6	6	6	-2	-2	-2	-2	-2	-2	-2	-2
χ_{14}	-2	-2	-2	-2	-2	0	0	0	0	-2	-2	-2	-2
χ_{15}	-3	1	-3	1	1	3	-3	1	-1	1	1	1	-1
χ_{16}	-3	1	-3	1	1	3	-3	1	-1	1	1	1	-1
χ_{17}	4	4	-4	-4	4	3	-3	1	-1	0	0	0	0
χ_{18}	4	4	-4	-4	4	3	-3	1	-1	0	0	0	0
χ_{19}	1	5	-7	-3	5	-3	3	-1	1	1	1	1	-1
χ_{20}	1	5	-7	-3	5	-3	3	-1	1	1	1	1	-1
χ_{21}	14	6	-2	-10	6	0	0	0	0	-2	-2	-2	2
χ_{22}	-13	-1	-5	7	-1	0	0	0	0	-1	-1	-1	1
χ_{23}	3	-1	3	-1	-1	0	0	0	0	3	3	3	-3
χ_{24}	-13	-1	-5	7	-1	0	0	0	0	-1	-1	-1	1
χ_{25}	3	-1	3	-1	-1	0	0	0	0	3	3	3	-3
χ_{26}	6	-2	6	-2	-2	0	0	0	0	-2	-2	-2	2
χ_{27}	-6	-2	2	-2	-2	3	3	-1	-1	2	2	-2	0
χ_{28}	-6	-2	2	-2	-2	3	3	-1	-1	2	2	-2	0
χ_{29}	10	-2	2	-2	-2	3	3	-1	-1	-2	-2	2	0
χ_{30}	10	-2	2	-2	-2	3	3	-1	-1	-2	-2	2	0
χ_{31}	4	-4	4	-4	-4	-3	-3	1	1	0	0	0	0
χ_{32}	4	-4	4	-4	-4	-3	-3	1	1	0	0	0	0
χ_{33}	14	10	-2	2	-6	0	0	0	0	-2	-2	2	0
χ_{34}	-2	10	-2	2	-6	0	0	0	0	2	2	-2	0
χ_{35}	-2	10	-2	2	-6	0	0	0	0	2	2	-2	0
χ_{36}	14	10	-2	2	-6	0	0	0	0	-2	-2	2	0
χ_{37}	28	-12	-4	4	4	0	0	0	0	4	4	-4	0
χ_{38}	-4	-12	-4	4	4	0	0	0	0	-4	-4	4	0
χ_{39}	-20	4	4	-4	4	0	0	0	0	0	0	0	0
χ_{40}	-20	4	4	-4	4	0	0	0	0	0	0	0	0

Table 7: Continued.

χ_{41}	8	0	0	0	0	1	-1	-1	1	4	-4	0	0
χ_{42}	8	0	0	0	0	1	-1	-1	1	4	-4	0	0
χ_{43}	8	0	0	0	0	1	-1	-1	1	4	-4	0	0
χ_{44}	8	0	0	0	0	1	-1	-1	1	4	-4	0	0
χ_{45}	16	0	0	0	0	2	-2	-2	2	-8	8	0	0
χ_{46}	-16	0	0	0	0	2	-2	-2	2	0	0	0	0
χ_{47}	-16	0	0	0	0	2	-2	-2	2	0	0	0	0
χ_{48}	0	0	0	0	0	-1	1	1	-1	0	0	0	0
χ_{49}	0	0	0	0	0	-1	1	1	-1	0	0	0	0
			4*B*					4*C*				5*A*	
	4*L*	4*M*	8*C*	8*D*	4*N*	4*O*	8*E*	8*F*	8*G*	8*H*	5*A*	10*A*	
χ_{1}	1	1	1	1	1	1	1	1	1	1	1	1	
χ_{2}	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	
χ_{3}	0	0	0	0	-2	-2	-2	-2	-2	-2	0	0	
χ_{4}	0	0	0	0	2	2	2	2	2	2	0	0	
χ_{5}	-1	-1	-1	-1	1	1	1	1	1	1	0	0	
χ_{6}	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	0	
χ_{7}	1	1	1	1	1	1	1	1	1	1	0	0	
χ_{8}	1	1	1	1	1	1	1	1	1	1	0	0	
χ_{9}	1	1	1	1	-1	-1	-1	-1	-1	-1	0	0	
χ_{10}	1	1	1	1	-1	-1	-1	-1	-1	-1	0	0	
χ_{11}	0	0	0	0	0	0	0	0	0	0	-1	-1	
χ_{12}	0	0	0	0	0	0	0	0	0	0	-1	-1	
χ_{13}	2	2	2	2	0	0	0	0	0	0	0	0	
χ_{14}	-2	-2	-2	-2	0	0	0	0	0	0	1	1	
χ_{15}	1	1	1	-1	3	-1	1	-3	-1	1	1	-1	
χ_{16}	1	1	1	-1	-3	1	-1	3	1	-1	1	-1	
χ_{17}	0	0	0	0	2	2	-2	-2	-2	2	-1	1	
χ_{18}	0	0	0	0	-2	-2	2	2	2	-2	-1	1	
χ_{19}	1	1	1	-1	1	-3	3	-1	1	-1	0	0	
χ_{20}	1	1	1	-1	-1	3	-3	1	-1	1	0	0	
χ_{21}	-2	-2	-2	2	0	0	0	0	0	0	1	-1	
χ_{22}	-1	-1	-1	1	-1	3	-3	1	-1	1	0	0	
χ_{23}	-1	-1	-1	1	3	-1	1	-3	-1	1	0	0	
χ_{24}	-1	-1	-1	1	1	-3	3	-1	1	-1	0	0	
χ_{25}	-1	-1	-1	1	-3	1	-1	3	1	-1	0	0	
χ_{26}	2	2	2	-2	0	0	0	0	0	0	0	0	
χ_{27}	2	2	-2	0	4	0	-2	2	0	-2	0	0	
χ_{28}	2	2	-2	0	-4	0	2	-2	0	2	0	0	
χ_{29}	-2	-2	2	0	-2	2	0	-4	2	0	0	0	
χ_{30}	-2	-2	2	0	2	-2	0	4	-2	0	0	0	
χ_{31}	0	0	0	0	-2	-2	2	2	-2	2	0	0	
χ_{32}	0	0	0	0	2	2	-2	-2	2	-2	0	0	
χ_{33}	2	2	-2	0	0	-4	-2	2	0	2	0	0	
χ_{34}	-2	-2	2	0	2	-2	-4	0	2	0	0	0	
χ_{35}	-2	-2	2	0	-2	2	4	0	-2	0	0	0	
χ_{36}	2	2	-2	0	0	4	2	-2	0	-2	0	0	

Table 7: Continued.

χ_{37}	0	0	0	0	0	0	0	0	0	0	0	0
χ_{38}	0	0	0	0	0	0	0	0	0	0	0	0
χ_{39}	0	0	0	0	2	2	2	2	-2	-2	0	0
χ_{40}	0	0	0	0	-2	-2	-2	-2	2	2	0	0
χ_{41}	4	-4	0	0	0	0	0	0	0	0	0	0
χ_{42}	-4	4	0	0	0	0	0	0	0	0	0	0
χ_{43}	4	-4	0	0	0	0	0	0	0	0	0	0
χ_{44}	-4	4	0	0	0	0	0	0	0	0	0	0
χ_{45}	0	0	0	0	0	0	0	0	0	0	0	0
χ_{46}	0	0	0	0	0	0	0	0	0	0	0	0
χ_{47}	0	0	0	0	0	0	0	0	0	0	0	0
χ_{48}	0	0	0	0	0	0	0	0	0	0	0	0
χ_{49}	0	0	0	0	0	0	0	0	0	0	0	0
			$6A$		$7A$	$7B$			$8A$		$14A$	$14B$
	$6D$	$6E$	$12A$	$12B$	$7A$	$7B$	$8I$	$8J$	$16A$	$16B$	$14A$	$14B$
χ_{1}	1	1	1	1	1	1	1	1	1	1	1	1
χ_{2}	-1	-1	-1	-1	1	1	-1	-1	-1	-1	-1	-1
χ_{3}	0	0	0	0	-1	-1	0	0	0	0	1	1
χ_{4}	0	0	0	0	-1	-1	0	0	0	0	-1	-1
χ_{5}	-1	-1	-1	-1	0	0	1	1	1	1	0	0
χ_{6}	1	1	1	1	0	0	-1	-1	-1	-1	0	0
χ_{7}	0	0	0	0	A	B	-1	-1	-1	-1	$-A$	$-B$
χ_{8}	0	0	0	0	B	A	-1	-1	-1	-1	$-B$	$-A$
χ_{9}	0	0	0	0	A	B	1	1	1	1	A	B
χ_{10}	0	0	0	0	B	A	1	1	1	1	B	A
χ_{11}	-1	-1	-1	-1	1	1	0	0	0	0	1	1
χ_{12}	1	1	1	1	1	1	0	0	0	0	-1	-1
χ_{13}	0	0	0	0	0	0	0	0	0	0	0	0
χ_{14}	0	0	0	0	0	0	0	0	0	0	0	0
χ_{15}	1	-1	-1	1	0	0	1	1	-1	-1	0	0
χ_{16}	-1	1	1	-1	0	0	-1	-1	1	1	0	0
χ_{17}	-1	1	1	-1	0	0	0	0	0	0	0	0
χ_{18}	1	-1	-1	1	0	0	0	0	0	0	0	0
χ_{19}	-1	1	1	-1	0	0	1	1	-1	-1	0	0
χ_{20}	1	-1	-1	1	0	0	-1	-1	1	1	0	0
χ_{21}	0	0	0	0	0	0	0	0	0	0	0	0
χ_{22}	0	0	0	0	0	0	1	1	-1	-1	0	0
χ_{23}	0	0	0	0	0	0	-1	-1	1	1	0	0
χ_{24}	0	0	0	0	0	0	-1	-1	1	1	0	0
χ_{25}	0	0	0	0	0	0	1	1	-1	-1	0	0
χ_{26}	0	0	0	0	0	0	0	0	0	0	0	0

Table 7: Continued.

χ_{27}	1	1	-1	-1	0	0	0	0	0	0	0	0
χ_{28}	-1	-1	1	1	0	0	0	0	0	0	0	0
χ_{29}	1	1	-1	-1	0	0	0	0	0	0	0	0
χ_{30}	-1	-1	1	1	0	0	0	0	0	0	0	0
χ_{31}	1	1	-1	-1	0	0	0	0	0	0	0	0
χ_{32}	-1	-1	1	1	0	0	0	0	0	0	0	0
χ_{33}	0	0	0	0	0	0	0	0	0	0	0	0
χ_{34}	0	0	0	0	0	0	0	0	0	0	0	0
χ_{35}	0	0	0	0	0	0	0	0	0	0	0	0
χ_{36}	0	0	0	0	0	0	0	0	0	0	0	0
χ_{37}	0	0	0	0	0	0	0	0	0	0	0	0
χ_{38}	0	0	0	0	0	0	0	0	0	0	0	0
χ_{39}	0	0	0	0	0	0	0	0	0	0	0	0
χ_{40}	0	0	0	0	0	0	0	0	0	0	0	0
χ_{41}	1	-1	1	-1	0	0	2	-2	0	0	0	0
χ_{42}	-1	1	-1	1	0	0	2	-2	0	0	0	0
χ_{43}	-1	1	-1	1	0	0	-2	2	0	0	0	0
χ_{44}	1	-1	1	-1	0	0	-2	2	0	0	0	0
χ_{45}	0	0	0	0	0	0	0	0	0	0	0	0
χ_{46}	0	0	0	0	0	0	0	0	C	$-C$	0	0
χ_{47}	0	0	0	0	0	0	0	0	$-C$	C	0	0
χ_{48}	1	-1	1	-1	0	0	0	0	0	0	0	0
χ_{49}	-1	1	-1	1	0	0	0	0	0	0	0	0

$A = (-1 - \sqrt{7}i)/2$, $B = (-1 + \sqrt{7}i)/2$, and $C = 2\sqrt{2}i$.

Table 8: The power maps of the elements of $2^9:(L_3(4):2)$.

$[g]_G$	$[x]_{\overline{G}}$	2	3	5	7	$[g]_G$	$[x]_{\overline{G}}$	2	3	5	7
$1A$	$1A$					$2A$	$2D$	$1A$			
	$2A$	$1A$					$2E$	$1A$			
	$2B$	$1A$					$4A$	$2B$			
	$2C$	$1A$					$4B$	$2B$			
							$4C$	$2B$			
$2B$	$2F$	$1A$				$3A$	$3A$		$1A$		
	$2G$	$1A$					$6A$	$3A$	$2C$		
	$4D$	$2A$					$6B$	$3A$	$2A$		
	$4E$	$2A$					$6C$	$3A$	$2B$		
	$4F$	$2B$									
	$4G$	$2B$									
	$4H$	$2B$									
	$4I$	$2B$									
$4A$	$4J$	$2F$				$4B$	$4L$	$2F$			
	$4K$	$2F$					$4M$	$2F$			
	$8A$	$4E$					$8C$	$4E$			
	$8B$	$4F$					$8D$	$4I$			
$4C$	$4N$	$2F$				$5A$	$5A$			$1A$	
	$4O$	$2F$					$10A$	$5A$		$2A$	
	$8E$	$4F$									
	$8F$	$4F$									
	$8G$	$4D$									
	$8H$	$4F$									
$6A$	$6D$	$3A$	$2D$			$7A$	$7A$			$1A$	
	$6E$	$3A$	$2E$								
	$12A$	$6C$	$4C$								
	$12B$	$6C$	$4A$								
$7B$	$7B$				$1A$	$8A$	$8I$	$4J$			
							$8J$	$4J$			
							$16A$	$8A$			
							$16B$	$8A$			
$14A$	$14A$	$7A$			$2D$	$14B$	$14B$	$7B$			$2D$

$$\theta_3 = \sum_{j=23}^{232} \varphi_j, \quad \deg(\theta_3) = 210$$

$$\theta_4 = \sum_{j=233}^{512} \varphi_j, \quad \deg(\theta_4) = 280. \tag{14}$$

Hence

$$\phi_{2^9} = a_1\varphi_1 + a_2\sum_{j=2}^{22}\varphi_j + a_3\sum_{j=23}^{232}\varphi_j + a_4\sum_{j=233}^{512}\varphi_j, \tag{15}$$

and therefore

$$\begin{aligned}\langle \phi_{2^9}, \phi_{2^9}\rangle &= a_1^2 + 21a_2^2 + 210a_3^2 + 280a_4^2 \\ &= \frac{1}{512}\{\phi(1A)\phi(1A) + 21\phi(2A)\phi(2A) \\ &\quad + 210\phi(2B)\phi(2B) + 280\phi(2C)\phi(2C)\},\end{aligned} \tag{16}$$

where $a_1 = \langle \zeta, \phi\rangle_{2^9:(L_3(4):2)}$.

We apply the above results to some of the irreducible characters of $U_6(2):2$ of small degrees, which in this case are $\phi_1 = 22a$, $\phi_2 = 22b$, $\phi_3 = 231a$, $\phi_4 = 231b$, $\phi_5 = 440a$, and $\phi_6 = 440b$. Their respective degrees are 22, 22, 231, 231, 440, and 440. For ϕ_1 we calculate that

$$\begin{aligned}&\langle \zeta, \phi_1\rangle_{2^9:(L_3(4):2)} \\ &\quad = \frac{1}{512}\{22 + 21(-10) + 210(6) + 280(-2)\} = 1.\end{aligned} \tag{17}$$

Now $a_1 + 21a_2 + 210a_3 + 280a_4 = 22$, since $\deg\phi_1 = 22$. Since $a_1 = 1$, we must have that $a_2 = 1$ and $a_3 = a_4 = 0$. Note that $2^9:(L_3(4):2)$ does not have irreducible characters of degree 22. We obtain that $(\phi_1)_{2^9:(L_3(4):2} = \chi_1 + \chi_{15}$ if the partial fusion of $2^9:(L_3(4):2)$ into $U_6(2):2$ is taken into consideration. Similarly, for ϕ_3 and ϕ_5 we calculate that

$$\begin{aligned}&\langle \zeta, \phi_3\rangle_{2^9:(L_3(4):2)} \\ &\quad = \frac{1}{512}\{231 + 21(39) + 210(7) + 280(-9)\} = 0\end{aligned} \tag{18}$$

and

$$\begin{aligned}&\langle \zeta, \phi_5\rangle_{2^9:(L_3(4):2)} \\ &\quad = \frac{1}{512}\{440 + 21(120) + 210(24) + 280(8)\} = 20.\end{aligned} \tag{19}$$

Since the respective degrees of ϕ_3 and ϕ_5 are 231 and 440, we have to solve the equations (i) $a_1 + 21a_2 + 210a_3 + 280a_4 = 231$ and (ii) $a_1 + 21a_2 + 210a_3 + 280a_4 = 440$, separately. If we are taking into account the fact that the set $Irr(\overline{G})$ (see Table 7) does not have any irreducible characters of degrees 231 and 440 and also that $\langle\zeta, \phi_3\rangle_{2^9:(L_3(4):2)} = 0$ and $\langle\zeta,\phi_5\rangle_{2^9:(L_3(4):2)} = 20$, we deduce that the two sets of values $\{a_1 = a_4 = 0, a_2 = a_3 = 1\}$ and $\{a_1 = 20, a_2 = 10, a_3 = 1, a_4 = 0\}$ are the only possibilities that satisfy equation (i) and (ii), respectively, hence we obtained that $(\phi_3)_{2^9:(L_3(4):2)} = \chi_{16} + \chi_{27}$ and $(\phi_5)_{2^9:(L_3(4):2)} = \chi_4 + \chi_{32}$. Similar computations were carried out to restrict the characters ϕ_2, ϕ_4, and ϕ_6 to $\overline{G}$ and we found that $(\phi_2)_{2^9:(L_3(4):2)} = \chi_2 + \chi_{16}$, $(\phi_4)_{2^9:(L_3(4):2)} = \chi_{15} + \chi_{28}$, and $(\phi_6)_{2^9:(L_3(4):2)} = \chi_3 + \chi_{31}$.

By making use of the values of ϕ_1, ϕ_2, ϕ_3, ϕ_4, ϕ_5, and ϕ_6 on the classes of $U_6(2):2$ and the values of $(\phi_1)_{2^9:(L_3(4):2)}$, $(\phi_2)_{2^9:(L_3(4):2)}$, $(\phi_3)_{2^9:(L_3(4):2)}$, $(\phi_4)_{2^9:(L_3(4):2)}$, $(\phi_5)_{2^9:(L_3(4):2)}$, and $(\phi_6)_{2^9:(L_3(4):2)}$ on the classes of $2^9:(L_3(4):2)$ together with the partial fusion, the complete fusion map of $2^9:(L_3(4):2)$ into $U_6(2):2$ is given in Table 9.

Data Availability

The data used to support the findings of this study are included within the article.

Table 9: The fusion of $2^9 : (L_3(4) : 2)$ into $U_6(2) : 2$.

$[g]_{L_3(4):2}$	$[x]_{2^9:(L_3(4):2)} \longrightarrow$	$[y]_{U_6(2):2}$	$[g]_{L_3(4):2}$	$[x]_{2^9:(L_3(4):2)} \longrightarrow$	$[y]_{U_6(2):2}$
$1A$	$1A$	$1A$	$2A$	$2D$	$2D$
	$2A$	$2A$		$2E$	$2E$
	$2B$	$2B$		$4A$	$4G$
	$2C$	$2C$		$4B$	$4H$
				$4C$	$4I$
$2B$	$2F$	$2B$	$3A$	$3A$	$3C$
	$2G$	$2C$		$6A$	$6G$
	$4D$	$4A$		$6B$	$6E$
	$4E$	$4B$		$6C$	$6F$
	$4F$	$4C$			
	$4G$	$4E$			
	$4H$	$4F$			
	$4I$	$4D$			
$4A$	$4J$	$4C$	$4B$	$4L$	$4D$
	$4K$	$4F$		$4M$	$4F$
	$8A$	$8A$		$8C$	$8A$
	$8B$	$8B$		$8D$	$8C$
$4B$	$4N$	$4H$	$5A$	$5A$	$5A$
	$4O$	$4I$		$10A$	$10A$
	$8E$	$8F$			
	$8F$	$8G$			
	$8G$	$8D$			
	$8H$	$8E$			
$6A$	$6D$	$6K$	$7A$	$7A$	$7A$
	$6E$	$6L$			
	$12A$	$12K$			
	$12B$	$12J$			
$7B$	$7B$	$7A$	$8A$	$8I$	$8E$
				$8J$	$8F$
				$16A$	$16A$
				$16B$	$16B$
$14A$	$14A$	$14A$	$14B$	$14B$	$14A$

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

The second author acknowledges the financial support provided by the South African Department of Defense and the academic support from the 1st author towards the completion of his M.S. degree at the University of the Western Cape.

References

[1] J. H. Conway, R. T. Curtis, S. P. Norton, R. A. Parker, and R. A. Wilson, *Atlas of Finite Groups*, Oxford University Press, Oxford, UK, 1985.

[2] I. A. I. Suleiman and R. A. Wilson, "Covering and automorphism groups of $U_6(2)$," *Quarterly Journal of Mathematics*, vol. 48, no. 192, pp. 511–517, 1997.

[3] C. Parker and G. Stroth, "An identification theorem for $PSU_6(2)$ and its automorphism group," 2011, https://arxiv.org/abs/1102.5392.

[4] The GAP Group, "GAP –groups, algorithms, and programming," 2013, http://www.gap-system.org.

[5] B. Fischer, "Clifford-matrices," in *Progress in Mathematics*, G. O. Michler and C. Ringel, Eds., pp. 1–16, Birkhauser, Basel, Switzerland, 1991.

[6] A. B. M. Basheer and J. Moori, "a survey on Clifford-Fischer theory," in *London Mathematical Society Lecture Notes Series*, vol. 422, pp. 160–172, Cambridge University Press, 2015.

[7] R. L. Fray, R. L. Monaledi, and A. L. Prins, "Fischer-Clifford matrices of 2^8:($U_4(2{:}2)$ as a subgroup of $O_{10}^{+}(2)$," *Afrika Matematika*, vol. 27, no. 7-8, pp. 1295–1310, 2016.

[8] J. Moori, "On certain groups associated with the smallest Fischer group," *Journal of the London Mathematical Society*, vol. 23, no. 1, pp. 61–67, 1981.

[9] J. Moori and Z. E. Mpono, "The Fischer-Clifford matrices of the group $2^6{:}SP_6(2)$," *Quaestiones Mathematicae*, vol. 22, pp. 257–298, 1999.

[10] A. L. Prins, "The Fischer-Clifford matrices and character table of the maximal subgroup $2^9{:}(L_3(4){:}S_3)$ of $U_6(2){:}S_3$," *Bulletin of the Iranian Mathematical Society*, vol. 42, no. 5, pp. 1179–1195, 2016.

[11] W. Bosma and J. J. Cannon, *Handbook of Magma Functions*, Department of Mathematics, University of Sydney, 1994.

[12] R. A. Wilson, P. Walsh, J. Tripp et al., *ATLAS of Finite Group Representations*, 1999, http://brauer.maths.qmul.ac.uk/Atlas/v3/.

[13] T. Connor and D. Leemans, "An atlas of subgroup lattices of finite almost simple groups," *Ars Mathematica Contemporanea*, vol. 8, no. 2, pp. 259–266, 2015.

[14] Z. Mpono, *Fischer-Clifford Theory and Character Tables of Group Extensions [PhD Thesis]*, University of Natal, 1998.

[15] J. J. Cannon, "An introduction to the group theory language CAYLEY," in *Computational Group Theory*, M. Atkinson, Ed., pp. 145–183, Academic Press, San Diego, Calif, USA, 1984.

[16] F. Ali, *Fischer-Clifford Theory for Split and Non-Split Group Extensions [Phd Thesis]*, University of Natal, 2001.

[17] D. Gorenstein, *Finite Groups*, Harper and Row Publishers, New York, NY, USA, 1968.

[18] T. T. Seretlo, *Fischer Clifford Matrices and Character Tables of Certain Groups Associated with Simple Groups O_{10}^{+}, HS and Ly [PhD Thesis]*, University of KwaZulu Natal, 2011.

[19] R. L. Fray and A. L. Prins, "On the inertia groups of the maximal subgroup $2^7{:}SP(6,2)$ in $\mathrm{Aut}(\mathrm{Fi}_{22})$," *Quaestiones Mathematicae*, vol. 38, no. 1, pp. 83–102, 2015.