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Comparison of the Lagrange's and Particle Swarm Optimisation Solutions of an Economic Emission Dispatch Problem with transmission constraints

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Abstract: The power demand is increased rapidly and hence the power systems become more complex, so it is necessary to solve the dispatch problem with less computation time. This paper uses the optimization approach (Lagrange's) and random variables selection approach Particle Swarm Optimisation (PSO) to solve the dispatch problem with transmission constraints and to compare the obtained solution and the time for its calculation. Formulation of a bi-criteria Combined Economic Emission Dispatch (CEED) problem is given. The application of Lagrange's and PSO methods to the CEED problem is described and the algorithms for calculations are given. The computational time of the Lagrange's algorithm depends on the selection of the initial values of the Lagrange's variable (λ), and on the swarms, positions, and velocity selection in PSO algorithm. The IEEE 30 bus system is considered to validate the simulation results in MATLAB environment. It concludes that Lagrange's algorithm provides better results for CEED problem in comparison to the PSO algorithm.

Keywords: Power System Optimization, Energy Management System, Economic Emission Dispatch, Lagrange's algorithm, Particle Swarm Optimization.

I. INTRODUCTION

The coal, petroleum and natural gas are the major sources of power production of the fossil plant. The US air clean act [4] amendments 1990 analyses how to reduce the waste and emissions such as N_2O, SO_2 and CO_2 from the fossil plants that create an environmental pollutions and green house effect. This research paper focuses on the Combined Economic Emission Dispatch (CEED) problem to minimize the fuel cost and emission of the fossil plants. The early work on the CEED problem is done in [1- 3]. In those papers the authors analyzed various methods for reducing emissions such as switching to low sulphur content coal, installing post-combustion cleaning system, and allocation of generation to each generator unit with the objective of minimum emission dispatch. Later the CEED problem is solved using price penalty factor applied in [5-7]. The CEED problem is solved using the Lagrange's algorithm in [8-9]. Particle Swarm Optimization is a population based stochastic optimization technique developed by Dr.Eberhart and Dr.Kennedy in 1995, inspired by social behavior of bird flocking or fish schooling [10]. The first work on the Particle Swarm Optimization algorithm to economic dispatch problem with prohibited operating zone is described in [11] and the first work on PSO solution to emission and economic dispatch problem is applied in [13]. The hybrid PSO-DE (Differential Evolution) algorithm is applied in [23] to solve the CEED problem but these types of hybrid algorithms use random variable selection approaches in both PSO and DE.

Both Lagrange's and PSO algorithms are generally used to solve the linear and non-linear dispatch optimization problems. The investigation in this paper focuses on comparison of the computational time used by both algorithms to calculate the optimal solution of the CEED problem. The dispatch problem is solved using both algorithms individually and the computational time used is compared. Section I presents the introduction, section II formulates the dispatch problem, section III formulates the bi-criteria dispatch problem, section IV and section V describe the application of the Lagrange's and PSO algorithm to the bi-criteria dispatch problem, section VI deals with description of the case studies and the results, sections VII and VIII present the discussion and conclusion.

II. ECONOMIC DISPATCH PROBLEM

The objective of the economic dispatch is to simultaneously minimize the generation cost rate and to meet the load demand of a power system over some appropriate period while satisfying various operating constraints. The objective function of an economic dispatch problem can be formulated as,

$$\text{Minimize } F_C = \sum_{i=1}^{n} F_i(P_i) = \sum_{i=1}^{n} (a_i P_i^2 + b_i P_i + c_i) [\$/hr] \quad (1)$$

Where
F_C - Total Fuel Cost
$F_i(P_i)$ - Fuel cost of the i^{th} generator
P_i - Real power generation of unit i
a_i, b_i, c_i - Cost coefficients of generating for unit i
n - Number of generating units
Under the constraints
1) Power balance constraint

$$\sum_{i=1}^{n} P_i = P_G = P_D + P_L [MW] \tag{2}$$

where

P_G - Total power generation of the system

P_D - Total demand of the system

P_L - Total transmission loss of the system

The transmission loss can be expressed as,

$$P_L = \sum_{i=1}^{n} \sum_{j=1}^{n} P_i B_{ij} P_j + \sum_{i=1}^{n} B_{0i} P_i + B_{00} \ [MW] \tag{3}$$

where

P_i - Active power generation of unit i

P_j - Active power generation of unit j

B_{ij}, B_{0i}, B_{oo} - Transmission loss coefficients

2) Generator operational constraints

$$P_{i,\min} \le P_i \le P_{i,\max}, i = \overline{1,n} \ [MW] \tag{4}$$

Where

$P_{i,\ min}$ - Minimum value of real power allowed at generator i

$P_{i,\ max}$ - Maximum value of real power allowed at generator i

III. Bi-CRITERIA DISPATCH PROBLEM

The various pollutants like sulphur dioxide, nitrogen oxide and carbon dioxide are the major waste emissions from the thermal power plants. The problem for minimization of the quantity of the emissions is formulated by including the reduction of emission as an objective by the following criterion

$$E_T = \sum_{i=1}^{n} (d_i P_i^2 + e_i P_i + f_i) \ [Kg/hr] \tag{5}$$

Where

E_T - Total emission

d_i, e_i, f_i - Emission coefficients of generating unit i

Then the Combined Economic Emission Dispatch (CEED) problem is determined by the objective functions (1) and (5).

A bi-objective optimization is converted into a single objective optimization problem by introducing price penalty factor h_i as follows,

$$F_T = \sum_{i=1}^{n} \left[(a_i P_i^{\ 2} + b_i P_i + c_i) + h_i \left(d_i P_i^2 + e_i P_i + f_i \right) \right] \ [\$/hr] \tag{6}$$

Where

F_T - CEED's fuel cost

$i = \overline{1,n}$

h_i are the penalty factors for i-th generator

If more pollutants are considered the criterion (5) can be expressed as

$$E_T = \sum_{i=1}^{n} \sum_{j=1}^{l} \left(d_{il} P_i^2 + e_{il} P_i + f_{il} \right)$$

Where

l is the number of the considered pollutants

Then the equation (6) is expressed as

$$F_T = \sum_{i=1}^{n} \left[(a_i P_i^{\ 2} + b_i P_i + c_i) + \sum_{j=1}^{l} h_{il} \left(d_{il} P_i^2 + e_{il} P_i + f_{il} \right) \right]$$

Single pollutant is considered in this paper.

The role of the penalty factor is to transfer the physical meaning of emission criterion from weight of the emission per hour to the fuel cost for the emission. Hence the unit of fuel cost is in [$/hr] and the unit of emission function is in [kg/hr]. The ratio between the fuel cost function and emission function is known as the price penalty factor in [$/kg]. It represents mathematical expression giving the ratio of cost criterion calculated for the minimum values of the real power and the total emission criterion calculated for the maximum values of the real power of the generators. This penalty factor is called Min-Max price penalty factor and is introduced in [25], Equation 7.

$$h_i = \frac{(a_i P_{i,\min}^2 + b_i P_{i,\min} + c_i)}{\left(d_i P_{i,\max}^2 + e_i P_{i,\max} + f_i \right)}, i = \overline{1,n} \quad [\$/kg] \tag{7}$$

IV. LAGRANGE'S METHOD FOR COMBINED ECONOMIC EMISSION DISAPTCH PROBLEM

Method of Lagrange's solution is applied to the CEED problem: criterion (6), subject to the constraints (2), (3) and (4). For solution of the problem a function of Lagrange is formulated by introduction of the Lagrange's multiplier λ , as follows:

$$L = F_T + \lambda \left(P_D + P_L - \sum_{i=1}^{n} P_i \right) \tag{8}$$

$$L = \left[\begin{array}{l} \sum_{i=1}^{n} \left[(a_i P_i^{\ 2} + b_i P_i + c_i) + h_i \left(d_i P_i^2 + e_i P_i + f_i \right) \right] + \\ + \lambda \left(P_D + \sum_{i=1}^{n} \sum_{j=1}^{n} P_i B_{ij} P_j + \sum_{i=1}^{n} B_{0i} P_i + B_{00} - \sum_{i=1}^{n} P_i \right) \end{array} \right] \tag{9}$$

Through the function of Lagrange the optimization criterion (6), subject to the constraints (2), (3) and (4) is transferred to the problem for minimization of L according to $P_{i,}$ $i = \overline{1,n}$, and maximization of L according the λ, under the constraints (4).

Necessary conditions for optimality for solution of the problem (4), (12) are,

According to P_i, $\quad \dfrac{\partial L}{\partial P_i} = 0, i = \overline{1,n}$ (10)

According to λ, $\dfrac{\partial L}{\partial \lambda}=0$ (11)

Derivation of the condition (10) is as follows:

$$\frac{\partial L}{\partial P_i}=2a_iP_i+b_i+h_i\left(2d_iP_i+e_i\right)+\lambda\left(2\sum_{j=1}^{n}B_{ij}P_j+B_{0i}-1\right)=0, i=\overline{1,n} \tag{12}$$

Equation (12) can be written in a matrix –vector form as follows

$$\text{EP=D} \tag{13}$$

Where

$$E=\begin{bmatrix}\frac{a_1+h_1d_1}{\lambda}+B_{11} & B_{12} & B_{1n}\\ B_{21} & \frac{a_2+h_2d_2}{\lambda}+B_{22} & B_{2n}\\ B_{n1} & B_{n2} & \frac{a_{1n}+h_nd_n}{\lambda}+B_{nn}\end{bmatrix}$$

$$D=\frac{1}{2}\begin{bmatrix}1-\left(\frac{b_1+h_1e_1}{\lambda}\right)-B_{01}\\ 1-\left(\frac{b_1+h_1e_1}{\lambda}\right)-B_{02}\\ 1-\left(\frac{b_1+h_1e_1}{\lambda}\right)-B_{0n}\end{bmatrix}$$

$$P=\begin{bmatrix}P_1 & P_2 & P_3 & \cdots P_n\end{bmatrix}^T \tag{14}$$

If the value of the Lagrange's multiplier λ is known the equation (17), can be solved according to the unknown vector P, using the Matlab command

$$P=E\setminus D \tag{15}$$

The value of λ is unknown and has to be found from the necessary condition for optimality, i.e. equation (11)

This condition is:

$$\frac{\partial L}{\partial \lambda}=\left(P_D+\sum_{i=1}^{n}\sum_{j=1}^{n}P_iB_{ij}P_j+\sum_{i=1}^{n}B_{0i}P_i+B_{00}-\sum_{i=1}^{n}P_i=0=\Delta\lambda\right) \tag{16}$$

Equation (19) is not a function of λ, but represents the gradient of L according to λ. At the optimal solution this gradient has to be equal to zero. As analytical solution for λ is not possible the gradient procedure for calculation of λ has to be developed as follows:

$$\lambda^{(k+1)}=\lambda^{k}+\alpha\Delta\lambda^{(k)}, \lambda\neq 0 \tag{17}$$

Where $\Delta\lambda^{(k)}$ is determined by the equation (16) and α is the step of the gradient procedure that starts with some given initial value of the Lagrange's variable $\lambda^{(0)}$. When during the iterations $\Delta\lambda=0$, the optimal solution for the Lagrange's variable is obtained. It will determine the optimal solution for the energy that has to be produced by the generators as a solution of (15).

The obtained solutions for $P_i,\ i=\overline{1,n}$ have to belong to the constraint domain (4). That is why for every step of the gradient procedure, the obtained solution is fit to the constraint domain following the procedure

$$P_i^{(k)}=\begin{Bmatrix}P_{i,\min}, & if & P_i^{(k)}{}_i\langle P_{i,\min}\\ P_i^{(k)}, & if & P_{i,\min}\leq P_i^{(k)}\leq P_{i,\max}\\ P_{i,\max}, & if & P_i^{(k)}\rangle P_{i,\max}\end{Bmatrix} \tag{18}$$

The condition for end of the iterations is

$$\Delta\lambda^{(k)}\leq\varepsilon, \text{ or } k=m \tag{19}$$

Where $\varepsilon\rangle 0$ is a small number used as criterion for completion of the gradient procedure and *m* is the given maximum number of iterations.

The algorithm of the method is:

1) Initial value of the Lagrange's multiplier is guessed: $\lambda^{(0)}$, and the value of the condition for optimality ε is given.
2) In equation (13) matrix $E^{(0)}$ and $D^{(0)}$ are formed
3) Equation (15) is solved and $P^{(0)}=E^{(0)}\setminus D^{(0)}$ is determined.
4) The obtained vector $P^{(0)}$ is fit to the constraints (18).
5) $\Delta\lambda^{(0)}$ is calculated using equation (16) where $P^{(0)}$ is substituted.
6) The condition (19) is checked. If it is fulfilled the calculations stop, if not, improved value of $\lambda\rightarrow\lambda^{(1)}$ is calculated using equation (17)
7) Calculations of the improved values of $P_i\rightarrow P_i^{(1)}$ is done as in equation (15) and so on. Iterations continue until condition (19) is satisfied or the maximum number of iterations is reached.
8) The optimal solution is used to calculate the total cost, using equation (6).

V. PSO ALGORITHM FOR SOLUTION OF THE CEED PROBLEM

A. Introduction of PSO Algorithm

Particle swarm optimization (PSO) is a population based stochastic optimization technique developed by Dr.Eberhart and Dr.Kennedy in 1995, inspired by social behavior of bird flocking or fish schooling [18]. PSO is initialized with a group of random particles and then searches for optimum solution by updating the position and velocity of the particles. In every iteration, each particle is updated by the following two best values. The first one is the best solution the particle has achieved so far. This value is called Pbest. Another best value that is tracked by the PSO is the best value, obtained so far by any particle in the population. This best value is a global best one and is called Gbest. After finding the two best values, the particle updates its velocity and positions according to the following equations (20) and (21).

$$V[] = \begin{bmatrix} \omega V[] + c1\,rand1()*\left(P^{best}[] - Ppresent[]\right) + \\ +c2\,rand2()\left(G^{best}[] - Ppresent[]\right) \end{bmatrix} \quad (20)$$

$$Ppresent[] = Ppresent[] + V[] \quad (21)$$

Where

V [ ] is the particle velocity

ω is the inertia weight and is calculated as,

$$\omega = \omega^{\max} - \left(\frac{\omega^{\max} - \omega^{\min}}{Iter^{\max}}\right) Iter$$

$$\omega^{\min} = 0.4\, and\, \omega^{\max} = 0.9$$

$Ppresent[]$ is the current particle

$P^{best}[]$ is the best solution of the particle of each iteration

$G^{best}[]$ is the best solution out of all the best particle's solutions

rand1( ) and *rand2( )* are the random numbers between (0,1)

c1 and *c2* are the learning factors (usually c1=c2=2)

B. Application of PSO algorithm to economic dispatch problem

A problem for optimization of the produced active power by *n* generators is considered. It is accepted that the number of generators in the electric power system is equal to the number of the members in a separate particle in the swarm. For the dispatch problem the positions of the member of the particles represent the active power produced by the generators. The velocities are variables that have the meaning of the active power but are used to do search in the constraints domain. It is assumed that the number of the particles in the swarm is *Np.* The algorithm for solution of the PSO problem is as follows:

Step1: Initialize the PSO parameters such as inertia weight $\omega^{\min}$ and $\omega^{\max}$, acceleration constants *c1* and *c2*, uniform random values *rand1* and *rand2*, maximum number of iterations $Iter^{max}$

Step2: Calculate the minimum and maximum velocity using equations (4) and (25) as follows:

$$-0.5P_{pi}^{\min} \le V_{pi} \le +0.5P_{pi}^{\max},\ p = \overline{1,N_p}, i = \overline{1,n} \quad (22)$$

Where

Np is the number of particles in the swarm

n is the number of members in the particles and is equal to the total number of generators

Step 3: Calculate the initial velocity of all members of the particles using equation (23)

$$V_{pi} = V_{pi}^{\min} + rand()\left(V_{pi}^{\max} - V_{pi}^{\min}\right),\ p = \overline{1,N_p}, i = \overline{1,n} \quad (23)$$

Where

$V_{pi}^{\min}, V_{pi}^{\max}$ are the previously calculated minimum and maximum velocities respectively

Step 4: Calculate the initial Position of the particles members using equation (24)

$$P_{pi} = P_{pi}^{\min} + rand()\left(P_{pi}^{\max} - P_{pi}^{\min}\right),\ p = \overline{1,N_p}, i = \overline{1,n} \quad (24)$$

Check the calculated positions within the limits as described

$$P_{pi} = \begin{Bmatrix} P_{pi}^{\min}, P_{pi} \le P_{pi}^{\min} \\ P_{pi}^{\max}, P_{pi} \ge P_{pi}^{\max} \\ P_{pi},\ \ P_{pi}^{\min} < P_{pi} < P_{pi}^{\max} \end{Bmatrix},\ p = \overline{1,N_p},\ i = \overline{1,n} \quad (25)$$

Step 5: The slack bus generator is considered as the dependent generator and is calculated using (26)

$$P_{pd} + \sum_{\substack{i=1\\ i\ne d}}^{n} P_i = \begin{bmatrix} \sum_{\substack{i=1\\ i\ne d}}^{n}\sum_{\substack{j=1\\ j\ne d}}^{n} P_{pi}B_{ij}P_{pj} + \sum_{\substack{j=1\\ j\ne d}}^{n} P_{pj}\left(B_{jd} + B_{dj}\right)P_{pd} + \\ +B_{dd}P_{pd}{}^2 + \sum_{\substack{i=1\\ i\ne d}}^{n} B_{io}P_{pi} + B_{do}P_{pd} + B_{oo} + P_D \end{bmatrix} \quad (26)$$

Where

$p = \overline{1,N_p}$

P_{pd} is the power produced by the slack bus generator

P_D is the total power demand of the system

The equation (26) can be simplified as [7] and is given in equation (27)

$$XP_{pd}^2 + YP_{pd} + Z = 0 \quad (27)$$

Where

$$X = B_{dd} \quad (28)$$

$$Y = \sum_{\substack{j=1\\ j\ne d}}^{n}\left(B_{jd} + B_{dj}\right)P_{pj} + B_{do} - 1 \quad (29)$$

$$Z = P_D + B_{oo} + \sum_{\substack{i=1\\ i\ne d}}^{n}\sum_{\substack{j=1\\ j\ne d}}^{n} P_{pi}B_{ij}P_{pj} + \sum_{\substack{i=1\\ i\ne d}}^{n} B_{io}P_{pi} - \sum_{\substack{i=1\\ i\ne d}}^{n} P_i \quad (30)$$

The positive roots of the equation are obtained as

$$P_{pd} = \frac{-Y \pm \sqrt{Y^2 - 4XZ}}{2X}, where Y^2 - 4XZ \geq 0 \tag{31}$$

$\sum_{i=1, i \neq d}^{n} P_i$ is the total active power of the system excluding the slack bus power.

Step 6: Calculate the following objective functions F_{Cp} for the initial position of the particles

i) Fuel cost function,

$$F_{Cp} = \sum_{i=1}^{n} (a_i P_{pi}{}^2 + b_i P_{pi} + c_i), p = \overline{1, N_p}$$

ii) Emission function,

$$E_{Tp} = \sum_{i=1}^{n} \left(d_i\, P_{pi}^2 + e_i\, P_{pi} + f_i \right), p = \overline{1, N_p}$$

iii) Combined economic emission function,

$$F_{Tp} = \sum_{i=1}^{n} \left[(a_i P_{pi}{}^2 + b_i P_{pi} + c_i) + h_{pi} \left(d_i\, P_{pi}^2 + e_i\, P_{pi} + f_i \right) \right], p = \overline{1, N_p}$$

Where the Min-Max price penalty factor is calculated as

$$h_{pi} = \frac{(a_i P_{pi,\min}^2 + b_i P_{pi,\min} + c_i)}{\left(d_i\, P_{pi,\max}^2 + e_i\, P_{pi,\max} + f_i \right)}, \ i = \overline{1, n}\ , \ p = \overline{1, N_p}$$

,

The fuel cost values of all the particles are sorted in ascending order. The first position value in the ascending order is selected as the F_T^{best}

Step 7: Select the best position and the global best position as follows:

i) Initial positions of particles in swarm are considered as best positions of particles P_{pi}^{best} $, p = \overline{1, N_p}, i = \overline{1, n}$

ii) The best position out of all the best particles P_{pi}^{best} is taken as G_{pi}^{best} $, p = \overline{1, N_p}, i = \overline{1, n}$

Step 8: Calculate new velocity using equation (32)

$$V_{pi}^{new} = w.V_{pi} + c1.rand1\left(P_{pi}^{best} - P_{pi} \right) + c2.rand2\left(G_{pi}^{best} - P_{pi} \right) \tag{32}$$

Check the constraints ,

$$V_{pi}^{new} = \begin{Bmatrix} V_{pi}^{new} > V_{pi}^{\max}, V_{pi}^{new} = V_{pi}^{\max} \\ V_{pi}^{new} < V_{pi}^{\min}, V_{pi}^{new} = V_{pi}^{\min} \end{Bmatrix}$$

If $V_{pi}^{new} > V_{pi}^{\max}, V_{pi}^{new} = V_{pi}^{\max}$ and

If $V_{pi}^{new} < V_{pi}^{\min}, V_{pi}^{new} = V_{pi}^{\min}$

Step 9: Calculate new particle position using equation (32)

$$P_{pi}^{new} = P_{pi} + V_{pi}^{new}\ , P = \overline{1, N_p}\ , i = \overline{1, n} \tag{33}$$

Step 10: Check the constraints (28)

Step 11: Calculate the new objective functions $F_T^{newbest}$ using step 6

Step 12: Check the new objective function $F_T^{newbest}$ as described below

$$\begin{aligned} &If\ F_T^{new} < F_T^{best}\ then\ F_T^{best} = F_T^{new} and\ P_{pi}^{best} = P_{pi}^{new} \\ &else\ F_T^{best} = F_T^{best} and\ P_{pi}^{best} = P_{pi}^{best} \end{aligned}$$

Step 13: Repeat the steps 5 to 12 until the maximum number of iterations is reached.

VI. CASE STUDY RESULTS

The algorithms described in part IV and V are applied to the solution of the CEED problem for IEEE 30 bus system. The data of fuel cost, emission coefficients and generator limits of IEEE 30 bus system are given in [9]. The various power demands of 150 MW to 250 MW is considered to solve the CEED problem. The calculations are carried out in the Matlab environment. The Lagrange's algorithm is solved for different initial lambda values of 4 and 10 respectively and its solution is given in Table 1. The PSO algorithm is solved for different particle values of 10 and 30 and its solution is given in Table 2 and Table 3 respectively.

VII. DISCUSSION

The optimized real powers of the generators are different in Lagrange's and PSO algorithms and is shown in Figure 5. The criterion function of the CEED problem is almost same and is shown in Figure 2 to 4. The computational time of the Lagrange's algorithm depends on the initial selection of the Lagrange's variable. If the initial value of the Lagrangian variable is close to its optimal value, then the computational time required to find the optimal solution is small. In vice-versa the solution space is enhanced and computational time required to find the optimal solution is bigger. In PSO algorithm, the computational time depends on the selection of number of particles. The selection of the smaller number of particles will narrow the solution search space and will require smaller computational time to find the near optimal solution and is shown in Figure1. In vice-versa the solution search space is enhanced and computational time required to find near to the optimal solution is bigger. The obtained optimal solution and computational time depend on the stopping criteria of Lagrange's and PSO algorithms which are tolerance error value and maximum number of iterations respectively. The future work of this paper will extends to hybrid Lagrange's–PSO algorithm where Lagrange's is the optimization approach and PSO is the random variable selection approach in order to combine the positive characteristics of the two algorithms.

The optimal solution of the CEED problem using Lagrange's algorithm is given in Table 1, where maximum number of iteration is 300 and CT denotes the calculation time.

Table 1: Application of Lagrange's algorithm for the bi-criteria dispatch problem solution

P_D [MW]	Optimal λ	P_1 [MW]	P_2 [MW]	P_3 [MW]	P_4 [MW]	P_5 [MW]	P_6 [MW]	P_L [MW]	F_C [$/hr]	E_T [Kg/hr]	F_T [$/hr]	$\lambda^0=4$		$\lambda^0=10$	
												CT [sec]	*Iter*	*CT [sec]*	*Iter*
150	3.0287	78.8314	26.2653	15.0000	10.0000	10.0000	12.0000	2.0967	373.5001	162.2299	448.5151	0.2174	153	0.2741	180
200	3.7044	114.0699	39.3656	18.8704	10.0000	10.0000	12.0000	4.3059	519.5033	228.1548	616.9529	0.1171	129	0.1929	166
250	4.1980	138.9676	48.7366	22.3455	18.8091	13.7142	13.8596	6.4336	687.0699	310.2823	815.1807	0.0751	85	0.1301	129

The best solution of the CEED problem using PSO algorithm is given in Table 2 and Table 3.

Table 2: Application of PSO algorithm for the bi-criteria dispatch problem solution

The Maximum number of iteration is 25, velocity is 5, and 10 particles are considered for the PSO method in Table 2

P_D [MW]	P_1 [MW]	P_2 [MW]	P_3 [MW]	P_4 [MW]	P_5 [MW]	P_6 [MW]	PL [MW]	F_C [$/hr]	E_T [Kg/hr]	F_T[$/hr]	Iteration Number	Best solution computation	*Overall Computation Time*
150	53.3617	33.3541	26.2260	15.6713	10.4352	12.4333	1.4815	392.6255	171.6735	471.1946	20	2.0074	2.5356
200	116.1479	29.2151	17.9788	11.2252	11.6081	17.9721	4.1471	524.8455	224.7032	623.3757	22	2.2682	2.8656
250	143.1200	47.6339	19.0569	18.7279	15.9568	12.1417	6.6372	686.0111	315.6620	816.5148	20	2.8225	2.9588

Table 3: Application of PSO algorithm for the bi-criteria dispatch problem solution

The Maximum number of iteration is 25, velocity is 5, and 30 particles are considered for the PSO method in Table 3

P_D [MW]	P_1 [MW]	P_2 [MW]	P_3 [MW]	P_4 [MW]	P_5 [MW]	P_6 [MW]	PL [MW]	F_C [$/hr]	E_T [Kg/hr]	F_T[$/hr]	Iteration Number	Best solution computation time [sec]	*Overall Computation Time [sec]*
150	63.9408	23.6347	27.7912	11.5311	10.3260	14.4160	1.6399	391.0849	165.8066	467.7885	4	2.0069	12.4357
200	102.4183	29.8400	24.4926	18.6308	14.0595	14.0026	3.4438	531.4352	214.1952	627.0118	7	2.8157	11.0558
250	143.0733	48.4094	20.2477	16.1364	11.1358	17.7469	6.7494	686.7437	316.3996	816.8095	2	0.8674	11.7269

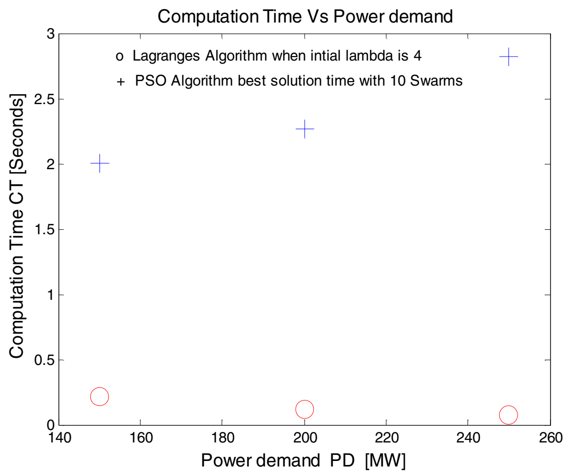

Figure 1 Computation time of Lagrange's and PSO algorithms for different cases of power demand

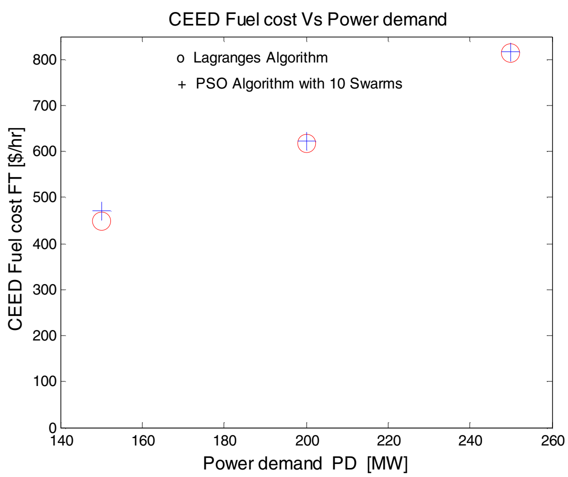

Figure 2 CEED fuel cost of Lagrange's and PSO algorithms for different cases of power demand

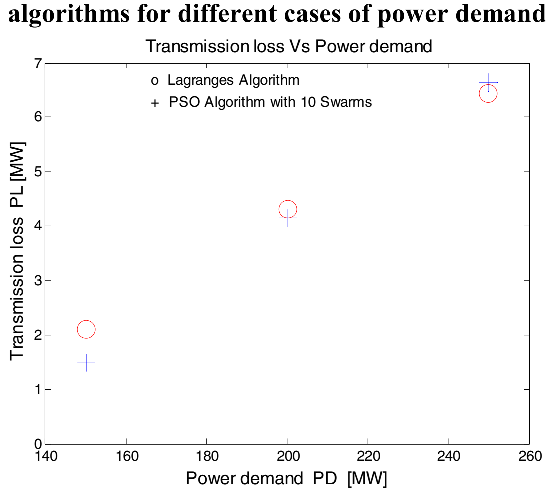

Figure 3 Transmission loss of Lagrange's and PSO algorithms for different cases of power demand

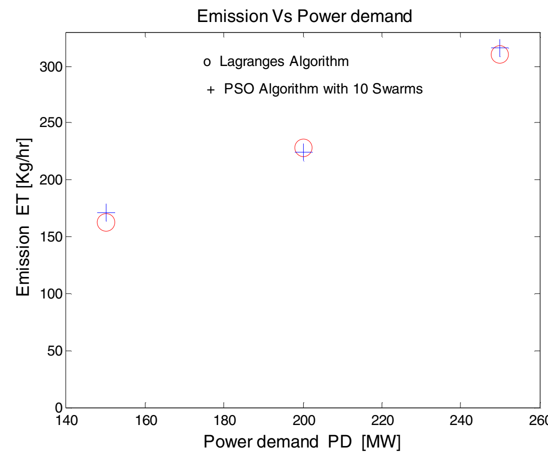

Figure 4 Emission values of Lagrange's and PSO algorithms for different cases of power demand

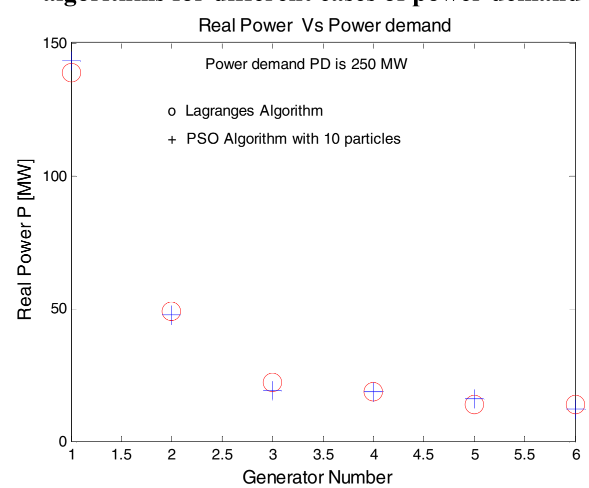

Figure 5 Generator real power values of Lagrange's and PSO algorithms for 250 MW power demand

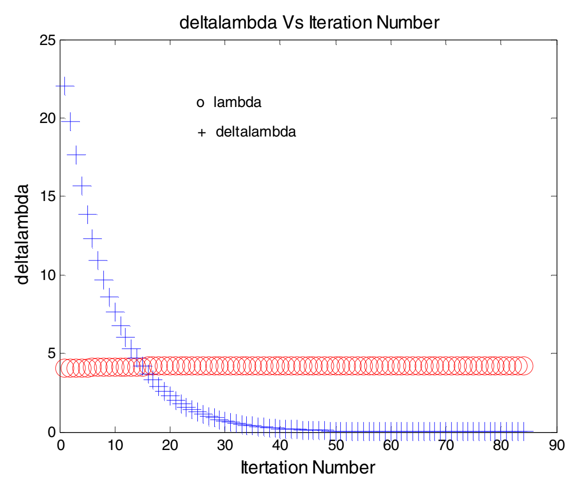

Figure 6 Optimized Lagrange variable lambda and delta lambda values over the iteration number for the power demand of 250 MW

VIII. CONCLUSION

The CEED problem with transmission constraints is solved using Particle Swarm Optimization and Lagrange's algorithm in this paper. The results of IEEE 30 bus system show that Lagrange's algorithm obtained optimal solution with less computational time and less CEED fuel cost value in comparison to the PSO algorithm. The transmission power loss and emission values are less in PSO algorithm. Finally it concludes that optimization approach (Lagrange's algorithm) provide better solution for CEED problem in compare to the random approach (PSO algorithm) as shown in Figures 1 to 4.

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