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Systematic properties of the Tsallis distribution: Energy dependence of parameters in high energy p–p collisions

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ABSTRACT

Changes in the transverse momentum distributions with beam energy are studied using the Tsallis distribution as a parameterization. The dependence of the Tsallis parameters q, T and the volume are determined as a function of beam energy. The Tsallis parameter q shows a weak but clear increase with beam energy with the highest value being approximately 1.15. The Tsallis temperature and volume are consistent with being independent of beam energy within experimental uncertainties.

1. Introduction

There exists a rich and wide variety of distributions covering a large range of applications [1–3]. Those having a power law behavior have attracted considerable attention in physics in recent years but there is a long history in other fields such as biology and economics [4].

In high energy physics power law distributions have been applied in [5–9] to the description of transverse momenta of secondary particles produced in p–p collisions. Indeed the available range of transverse momenta has expanded considerably with the advent of the Large Hadron Collider (LHC). Collider energies of 7 TeV are now available in p–p collisions and transverse momenta of hundreds of GeV are now common. Applications of the Tsallis distribution to high energy e^+–e^- annihilation has been considered previously in [10]. A recent review of power laws in elementary and heavy-ion collisions can be found in [11]. The Tsallis distribution has the advantage of being connected, via the entropy, to thermodynamics which is not the case for other power-law distributions [12,13]. In this Letter the focus is on the energy dependence of the Tsallis parameters describing the transverse momenta at mid-rapidity in high energy p–p collisions. The rapidity distributions contain more dynamics, e.g. strong longitudinal expansion, and will not be considered in this Letter which is limited to mid-rapidity.

2. Choice of distribution

In the analysis of the new data, a Tsallis-like distribution gives excellent fits to the transverse momentum distributions as shown by the STAR [5] and PHENIX [6] Collaborations at RHIC and by the ALICE [7], ATLAS [8] and CMS [9] Collaborations at the LHC. In this Letter we review the parameterization used by these groups and propose a slightly different one which leads to a more consistent interpretation and has the bonus of being thermodynamically consistent.

In the framework of Tsallis statistics [14–21] the particle number, N is are given by an integral over the Tsallis distribution

$$N = gV \int \frac{d^3p}{(2\pi)^3}\left[1+(q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}}, \tag{1}$$

where T and μ are the temperature and the chemical potential, V is the volume and g is the degeneracy factor.

Note the extra power of q compared to other forms of the Tsallis distribution, this is necessary in order to have thermodynamic consistency. The motivation for using the above expressions has been presented in detail in [16,19]. Other proposals have been made in the literature, see e.g. [22–24] and using a mean field approach [25]. In this case a slightly different form was used for the particle number

$$N = gV \int \frac{d^3p}{(2\pi)^3}\left[1+(q-1)\frac{E-\mu}{T}\right]^{-\frac{1}{q-1}}. \tag{2}$$

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Numerically the difference is not very large as the value of q is always close to 1. We have checked that the resulting values for q are slightly smaller in this case than those following from (1).

The particular form of the Tsallis distribution presented above has been referred to as Tsallis-B distribution in [19]; in [16] it was discussed in great detail for the Fermi–Dirac form of the Tsallis distribution. Following from (1), the momentum distribution is given by

$$\frac{d^3N}{d^3p} = \frac{gV}{(2\pi)^3}\left[1+(q-1)\frac{E-\mu}{T}\right]^{-q/(q-1)}, \tag{3}$$

or, expressed in terms of transverse momentum, p_T, the transverse mass, $m_T \equiv \sqrt{p_T^2+m^2}$, and the rapidity y

$$\frac{d^2N}{dp_T\,dy} = gV\frac{p_T m_T \cosh y}{(2\pi)^2} \times \left[1+(q-1)\frac{m_T\cosh y-\mu}{T}\right]^{-q/(q-1)}. \tag{4}$$

At mid-rapidity, $y=0$, and for zero chemical potential, as is relevant at the LHC, this reduces to

$$\left.\frac{d^2N}{dp_T\,dy}\right|_{y=0} = gV\frac{p_T m_T}{(2\pi)^2}\left[1+(q-1)\frac{m_T}{T}\right]^{-q/(q-1)}. \tag{5}$$

In the limit where the parameter q goes to 1 it is well known that this reduces the standard Boltzmann distribution:

$$\lim_{q\to 1}\frac{d^2N}{dp_T\,dy} = gV\frac{p_T m_T\cosh y}{(2\pi)^2}\exp\left(-\frac{m_T\cosh y-\mu}{T}\right). \tag{6}$$

The parameterization given in Eq. (4) is close to the one used e.g. by the ALICE and other Collaborations [5–9]:

$$\frac{d^2N}{dp_T\,dy} = p_T\frac{dN}{dy}\frac{(n-1)(n-2)}{nC(nC+m_0(n-2))}\left[1+\frac{m_T-m_0}{nC}\right]^{-n}, \tag{7}$$

where n and C are fit parameters. This formula can also be understood as an interpolation between low transverse momenta exponential fall-off and high momenta QCD power law behavior [12,13]. This corresponds to substituting

$$n \to \frac{q}{q-1}, \tag{8}$$

and

$$nC \to \frac{T+m_0(q-1)}{q-1}. \tag{9}$$

After this substitution Eq. (7) becomes

$$\frac{d^2N}{dp_T\,dy} = p_T\frac{dN}{dy}\frac{(n-1)(n-2)}{nC(nC+m_0(n-2))} \times \left[\frac{T}{T+m_0(q-1)}\right]^{-q/(q-1)} \times \left[1+(q-1)\frac{m_T}{T}\right]^{-q/(q-1)}. \tag{10}$$

At mid-rapidity $y=0$ and zero chemical potential, this has the same dependence on the transverse momentum as (5) apart from an additional factor m_T on the right-hand side of (4). However, the inclusion of the rest mass in the substitution (9) is not in agreement with the Tsallis distribution as it breaks m_T scaling which is present in Eq. (4) but not in Eq. (7). The inclusion of the factor m_T leads to a more consistent interpretation of the variables q and T.

It is to be noted the variables (T, V, q, μ) in the distribution function (3) have a redundancy for $\mu \neq 0$ and are not independent. Indeed, let $T = T_0$ and $V = V_0$ at $\mu = 0$ and fixed values of q. Comparing Eq. (3) written for $\mu = 0$ and the same equation written for finite values of μ, we obtain

$$T_0 = T\left[1-(q-1)\frac{\mu}{T}\right], \quad \mu \leqslant \frac{T}{q-1}, \tag{11}$$

$$V_0 = V\left[1-(q-1)\frac{\mu}{T}\right]^{\frac{q}{1-q}}. \tag{12}$$

Therefore, the variables T and V are functions of μ at fixed values of q and they can be calculated if the parameters (T_0, V_0) and q are known. This redundancy is not present when $\mu = 0$, which is the case relevant for the LHC due to the particle–antiparticle symmetry.

A very good description of transverse momenta distributions at RHIC has been obtained in Refs. [26,27] on the basis of a coalescence model where the Tsallis distribution is used for quarks. In this Letter we use the Tsallis distribution for hadrons, not for quarks.

A Tsallis fit has also been considered in Ref. [28] but a different power law in the Tsallis function was considered by these authors.

Interesting results were obtained in Refs. [29–32] where spectra for identified particles were analyzed and the resulting values for the parameters q and T were considered. These authors did not consider the energy dependence which is the main focus of the present Letter.

The transverse momentum distribution in connection with the multiplicity in different events was consider in Ref. [33].

The energy dependence of the transverse momentum spectra has been studied on the basis of gluon collisions in Ref. [34].

3. Energy dependence of transverse momentum distributions

The energy dependence in p–p collisions can be determined by studying data at beam energies of 0.54 [35], 0.9, 2.36 and 7 TeV [8,9]. These involve distributions summed over charged particles. The fits were performed using a sum of three Tsallis distributions, the first one for π^+, the second one for K^+ and the third one for protons p. The relative weights between these were simply determined by the corresponding degeneracy factors, i.e. 1 for π^+ and K^+ and 2 for protons. The fit was taken at mid-rapidity and for $\mu = 0$ using the following expression:

$$\left.\frac{1}{2\pi p_T}\frac{d^2N(\text{charged particles})}{dp_T\,dy}\right|_{y=0} = \frac{2V}{(2\pi)^3}\sum_{i=1}^{3} g_i m_{T,i}\left[1+(q-1)\frac{m_{T,i}}{T}\right]^{-\frac{q}{q-1}}, \tag{13}$$

where $i = (\pi^+, K^+, p)$ and $g_{\pi^+} = 1$, $g_{K^+} = 1$ and $g_p = 2$. The factor 2 in front of the right-hand side of this equation takes into account the contribution of the antiparticles $(\pi^-, K^-, \bar{p})$. The resulting values of the parameters are shown in Table 1.

The Tsallis parameter q has a tendency to increase slowly but clearly with increasing energy.

The radius is listed instead of the volume V in Eq. (13) and is defined as $R \equiv [V\frac{3}{4\pi}]^{1/3}$. Note that the volume can contain dynamical factors, e.g. for a cylindrical expansion along the beam axis the volume could be interpreted as the transverse surface multiplied by the lifetime, τ, of the system, e.g. $\pi R^2\tau$ as would be more appropriate for a scenario with longitudinal scaling [36]. The value of R should therefore not be considered in a full dynamical model. It is not necessarily related to the size of the system as deduced

Table 1
Parameters for the Tsallis fit.

$\sqrt{s}$ (TeV)	T (MeV)	R (fm)	q
0.54 (UA1)	77.59 ± 1.40	4.25 ± 0.10	1.1175 ± 0.0014
0.9 (ALICE)	75.45 ± 3.18	4.55 ± 0.26	1.1305 ± 0.0031
0.9 (ATLAS)	83.89 ± 1.35	3.91 ± 0.11	1.1217 ± 0.0007
2.36 (ATLAS)	75.79 ± 4.01	4.46 ± 0.37	1.1419 ± 0.0025
7 (ATLAS)	82.42 ± 1.30	4.34 ± 0.11	1.1479 ± 0.0008

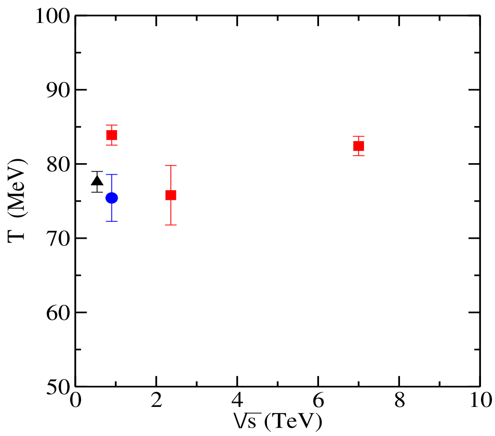

Fig. 1. (Color online.) Energy dependence of the Tsallis temperature T appearing in the Tsallis distribution. Square points are from the ATLAS Collaboration [8], the round point is from the ALICE Collaboration [7] and the triangle point is from the UA1 Collaboration [35].

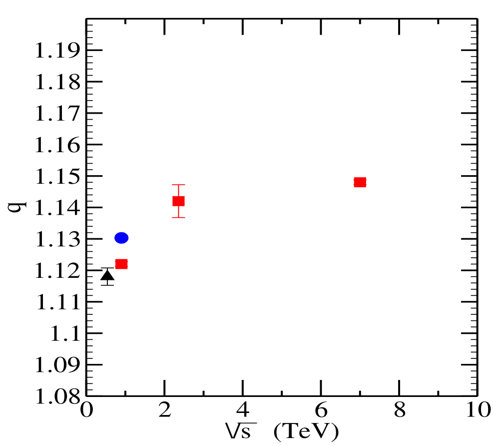

Fig. 2. (Color online.) Energy dependence of the Tsallis parameter q appearing in the Tsallis distribution. Square points are from the ATLAS Collaboration [8], the round point is from the ALICE Collaboration [7] and the triangle point is from the UA1 Collaboration [35].

from a HBT analysis [37–39] but serves to fix the normalization of the distribution. The radius (volume) and the temperature do not show any significant change between the lowest beam energy considered (0.54 TeV) and the highest one (7 TeV) as can be seen from Figs. 1 and 3.

The energy dependence of the various parameters is displayed in Figs. 1, 2 and 3.

In Fig. 4 we show the charged hadron yields as a function of the transverse momentum p_T at four different energies. The experimental data points are from the ATLAS Collaboration [8] for p–p collisions at energies $\sqrt{s} = 0.9,\ 2.36,\ 7$ TeV and from the UA1

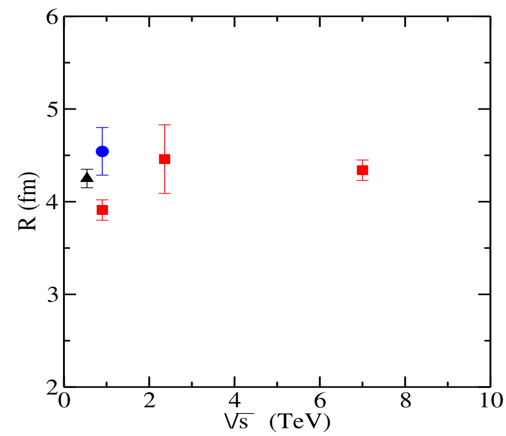

Fig. 3. (Color online.) Energy dependence of the Tsallis radius R appearing in the volume factor. Square points are from the ATLAS Collaboration [8], the round point is from the ALICE Collaboration [7] and the triangle point is from the UA1 Collaboration [35].

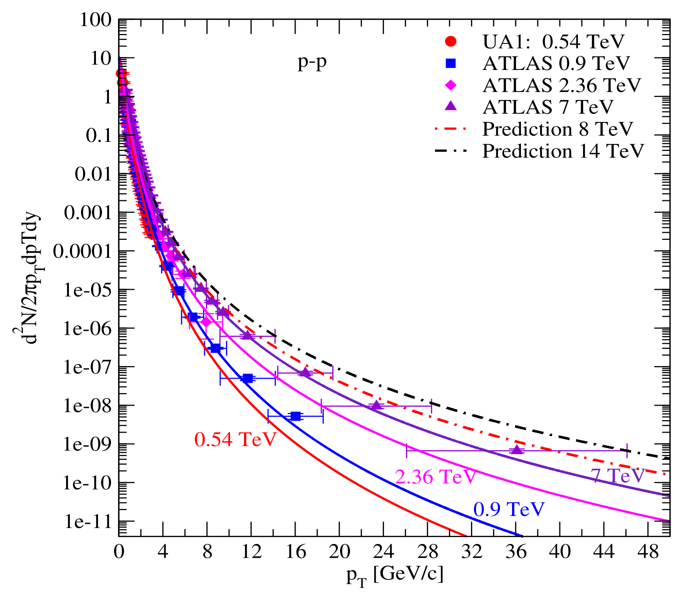

Fig. 4. (Color online.) Charged hadron yield as a function of the transverse momentum p_T from the UA1 Collaboration in $p\bar{p}$ collisions at $\sqrt{s} = 0.54$ TeV [35] and data from the ATLAS Collaboration [8]. The curves were obtained using Eq. (13). The parameters of the fit are given in Table 1.

Collaboration in p–$\bar{p}$ collisions at $\sqrt{s} = 0.54$ TeV [35]. The curves represent fits of Eq. (13) to the data.

The extremely large range of p_T described by Tsallis distribution makes it applicable in the region usually considered to be the domain of QCD hard scattering. This may be interpreted as a manifestation of the "duality" between the statistical and dynamical description of strong interactions [40,41]. In this sense Tsallis statistics may be considered as an effective theory allowing for an extension of the region of applicability of perturbative QCD from large to low p_T. It is not unnatural, as approximate scale invariance manifested in QCD both at large and small momentum scales is qualitatively similar to power law statistics. It remains to be understood whether any further relations can be found, like the dynamical origins of thermal spectra [42]. Note, that the application of the duality to the longitudinal spectra should be performed at the level of parton distributions, where the natural temperature scale is defined by the corresponding intrinsic transverse

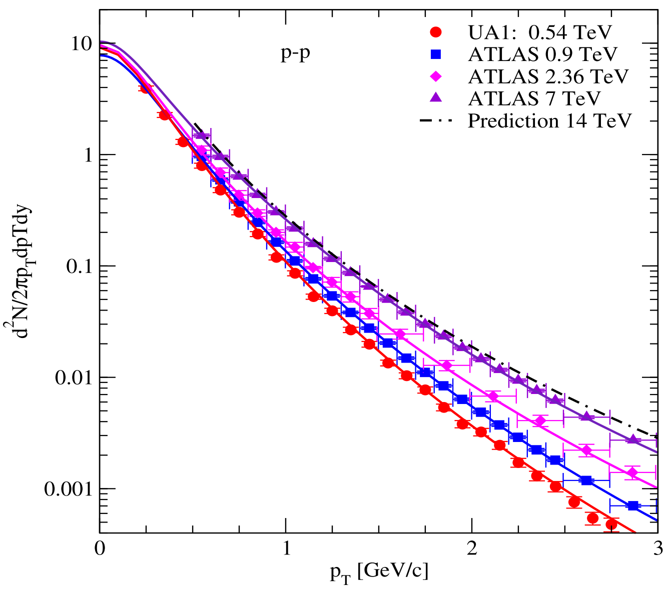

Fig. 5. (Color online.) Charged hadron yield as a function of the transverse momentum p_T from the UA1 Collaboration in $p\bar{p}$ collisions at $\sqrt{s} = 0.54$ TeV [35] and data from the ATLAS Collaboration [8] showing only the low transverse momentum part. The curves were obtained using Eq. (13). The parameters of the fit are given in Table 1.

momentum [43], rather than hadrons directly, which would lead to the "temperature" of beam energy scale order.

The Tsallis parameter q shows a clear but weak energy dependence which we have parameterized as

$$q(s) = 1.12567(\sqrt{s})^{0.011211}. \tag{14}$$

On the basis of these changes with energy we have attempted a prediction for the transverse momentum distribution at invariant energy of 8 and 14 TeV which are presented in Fig. 4. A different parameterization has been proposed by Wibig [44,45], the values obtained there are systematically below the values obtained here. The fits are also very good at low transverse momenta as can be seen from Fig. 5. The curves are calculations using Eq. (13) at mid-rapidity $y = 0$ and zero chemical potential $\mu = 0$ for the parameters of the Tsallis fit in Table 1.

4. Discussion and conclusions

As has been noticed in other publications, the Tsallis distribution leads to excellent fits to the transverse momentum distributions in high energy p–p collisions. By comparing results from UA1 [35] to results obtained at the LHC [7–9] it has been possible to extract the parameters q, T and V for a wide range of energies. A consistent picture emerges from a comparison of fits using the Tsallis distribution to high energy p–p collisions.

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