Laminar and Transitional Flow in open channels for non-Newtonian fluids.

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ABSTRACT

The laminar-turbulent transition for pipelines has been studied intensively and several models are available to predict transition with various degrees of success. Generally the Reynolds number of 2100 is used to describe transition in pipelines. Three smooth rectangular tilting flumes with flow widths of 75, 150 and 300 mm were used to test various concentrations of Kaolin and Bentonite suspensions as well as Carboxymethylcellulose (CMC) solutions. By using the appropriate Reynolds number, all the laminar data could be collapsed onto a standard Moody diagram for water. In laminar flow the friction factor can be predicted for all the materials tested. Transitional behaviour was observed not at one Reynolds number, but over a range. For the higher concentration fluids, transition occurred at lower Reynolds numbers and vice versa. This makes it very difficult to accurately predict the transition of non-Newtonian fluids in open channels.

KEY WORDS: open channel, flume, non-Newtonian, rheology, laminar flow, transition, turbulence, viscometry

NOTATION

A	-	area	m^2
D	-	diameter	m
De	-	equivalent diameter	m
f	-	Fanning friction factor	
K	-	consistency index	$Pa.s^n$
L	-	distance between pressure tappings	m
n	-	flow-behavior index	
n´	-	apparent flow behavior index	
Δp	-	change in pressure	Pa
Re	-	Reynolds number	

R_h	-	hydraulic radius	m
du/dr	-	(true shear rate)	1/s
V	-	average velocity	m/s
ρ	-	density	kg/m^3
μ	-	viscosity of the fluid	
τ_0	-	wall shear stress	Pa
τ_y	-	yield stress	Pa
θ	-	angle of flume above the horizontal	degrees

1. INTRODUCTION

When dealing with the flow of non-Newtonian fluids in open channels very little is found in the literature and textbooks have scant references to the same. In the past certain authors have tried to simplify the matter by suggesting that pipe flow design criteria can be adapted to accommodate open channel flow behavior. The Flow Process Research Centre of the Cape Technikon is conducting extensive tests to address some of the problems related to the flow of non-Newtonian suspensions in open channels. This paper will endeavor to show that laminar flow for non-Newtonian suspensions can be predicted for flow in smooth rectangular open channels when one is certain that the flow is laminar. The transition that for pipe flow, and smooth open channel flow for water, occurs at an approximate Reynolds number of 2100, behaves quite differently when dealing with viscous suspensions. Certain trends will be pointed out, although at this stage, we are not in a position to accurately predict transition.

2. LITERATURE

The laminar flow of different homogenous non-Newtonian slurries has been described in the literature by Kozicki and Tiu (7), Hao & Zhenghai (6), Coussot (3), De Kee *et al* (4) and Haldenwang *et al* (5). No data was produced by Kozicki and Tiu and by de Kee *et al*.

Kozicki and Tiu (7) give a wide range of velocity and Reynolds number formulae for different model fluids. Their work also includes the effect of different flume cross-sectional shapes on the flow. They do not verify their work with experimental data. For transition they refer to work done by Straub *et al* (11) on Newtonian fluids, that describes transition to occur between Re = 2000-3000. This they proposed mainly depended on the channel shape.

Hao & Zhenghai (6) tested Yellow river mud, which they classified as a Bingham fluid. They present some data but it is difficult to follow what the rheology of the data presented is. They found that at transition there is a pulsation of flow, which develops from the boundary layer upwards, until at full turbulence, the pulsation is the same at each level of the flow depth. They also found that the transition for smooth channels occurred at a reasonably constant Reynolds number. For different channel roughness', transition occurred at lower Reynolds numbers. The greater the roughness, the lower the Reynolds number at transition. They suggested that the channel roughness accelerated the development of turbulence in the boundary layer. The Reynolds number that they published for transition varied between 3000 and 5000 depending on the channel roughness size which varied from 0,3 mm to 7,6 mm. The data also shows a distinct jump in friction factor at transition.

Coussot (3) presents data, but only in the laminar region up to a Reynolds number of about 1000. The materials tested were at high concentrations and therefore with low Reynolds numbers. The data used by Coussot was tested with the Reynolds number used in this paper and the data fitted very well. Coussot mentions that the Hanks criterion for pipe flow could be suitable for the free surface flow of Bingham fluids after the necessary rheological transformation has been preformed. This method uses the Hedstrom number. For the pipe diameter one has to substitute $4R_h$. This presents a problem in that one has to combine slope, depth and width in the one number.
The test procedure used will be described next.

3. TEST PROCEDURES

The test equipment used consists of the following:

- A 10 m long by 300 mm wide rectangular tilting flume (maximum slope 5 degrees), which can be partitioned to make up a 150 mm wide flume. A 100 mm positive displacement pump and a 4x3 inch Warman centrifugal pump produce the flow.
- A 5 m long by 75 mm wide rectangular tilting flume (maximum slope 5 degrees) with flow supplied by the positive displacement pump.
- Both flumes are linked to an on-line tube viscometer with 3 diameter tubes namely 13 mm, 28 mm and 80 mm diameter.
- Each tube is linked to a high and a low range differential pressure gauge has a magnetic flow meter in line. The 13 mm tube also has a mass-flow meter, which can provide on-line temperature and density.
- The depth of flow in the channel is measured with electronic depth verniers, which are linked via interfaces to the computer. The two depth probes in the 10 m flume are 1 m apart at 5 and 6 m from the entrance to the flume. The difference in flow height at this position was less than 4%. From this it was concluded that the flow was developed.
- All the transducers are linked to an HP data logger, which is interfaced with a PC.

A diagrammatic layout of the 10 m flume rig is presented in Figure 1, and the pipe viscometer rig is presented in Figure 2. The 75 mm flume is separate, but attached to the pipe viscometer.

In order to analyze the flow, the rheology of the non-Newtonian fluid first has to be established. Using the relevant friction factor and Reynolds number all the data can be depicted on the standard Moody diagram. The following procedure was used to establish the on-line rheology:

- Calibration of instruments. The flow meters, differential pressure transducers, flow depth gauges, and load cell all have to be calibrated so that they can interface electronically with the computer.
- Water tests then have to be conducted. The flow characteristics of water are known and therefore the equipment can be checked.
- Different materials are then used to represent various rheological model fluids. The following suspensions were used in tests: Kaolin, Bentonite and CMC. Altogether 11 concentrations were tested. They can be classified as three rheologically different fluids. Kaolin is a Yield Pseudoplastic fluid, Bentonite a Bingham fluid and CMC a Power law or Pseudoplastic fluid.
- The fluids are then pumped through 3 different sized pipes and flow rate and pressure drop are measured over a large range of flow rates. The pressure drop is used to calculate the wall shear stress and the flow-rate to calculate the pseudo shear rate. This is then plotted on a pseudo shear diagram. The laminar flow data are co-linear and the turbulent data for each diameter pipe branch off on different lines.All the laminar tube data are then transformed by means of the Rabinowich Mooney transformation method to true shear rate using Equation (1):
  Where

$$\left(-\frac{du}{dr}\right)_0 = \left(\frac{3n'+1}{4n'}\right)\frac{8V}{D} \quad \text{and } n'= \frac{d\ln\frac{D\Delta p}{4L}}{d\ln\frac{8V}{D}} \tag{1}$$

- To the Rheogram data, a rheological model is then fitted and the rheological parameters obtained. This describes then the rheology of the material. The yield pseudoplastic model is used for the Kaolin (Equation 2).

$$\tau_0 = \tau_y + K\left(-\frac{du}{dr}\right)_0^n \tag{2}$$

- Using the same equation for CMC which is a Pseudo plastic material, the yield stress becomes 0, and for Bentonite which is a Bingham fluid n=1.
- The same fluid tested in the on-line pipe viscometer is then tested in three rectangular flumes at different slopes and over a wide range of flow rates.
- The fluid depth and flow rate is then measured.
- The friction factor and Reynolds number are then calculated and all the data are plotted on a Moody diagram. See equations 3 to 6.

$$f = \frac{g.De.\sin\theta}{2V^2} \qquad \text{(Fanning friction factor)} \quad \text{For flumes } De = 4R_h \tag{3}$$

$$\mathrm{Re} = \frac{\rho 8 V^2}{K\left(\frac{2V}{R_h}\right)^n + \tau_y} \tag{4}$$

For a Bingham fluid n=1 and for a pseudo plastic fluid $\tau_y = 0$

$$f = \frac{16}{\mathrm{Re}} \quad \text{(laminar flow)} \tag{5}$$

$$f = 0{,}079\mathrm{Re}^{-0{,}25} \quad \text{(turbulent flow)} \tag{6}$$

Equation 6 is the Blasius equation for turbulent flow of Newtonian fluids. Chow (2).
The Reynolds number used in Equation 4 is derived from the Re2 Reynolds number proposed by Slatter (8) for pipes. It is useful as it can be easily adapted for pseudoplastic and Bingham fluids.

4. RESULTS

The results discussed below will concentrate on the laminar flow region and laminar-turbulent transition.

Five concentrations of Kaolin suspension namely 3% 4,5% 6% 8% and 10% were tested in all three flumes at slopes varying from 1-5 degrees. The combined data for the 75 mm flume are depicted in Figure 7, for the 150 mm flume in Figure 8 and for the 300 mm flume in Figure 9.

Figure 10 shows the test data for 4 concentrations CMC in the 75 mm flume, Figure 11 the data in the 150 mm flume and Figure 12 the data in the 300 mm flume. The concentrations were 1%, 1,8%, 2,8% and 3,8%.

Figure 13 shows the test data for 3 concentrations Bentonite suspension in the 75 mm flume and Figure 14 the data in the 150 mm flume. The concentrations were 3% 4,5 % and 6%.

A summary of the fluid properties is given in Table 1 below. The concentrations were all by volume.

Table 1. Fluid Properties of the Materials Tested

Material	Concentration By volume	Density kg/m^3	τ_y Pa	K Pa.s^n	n
Kaolin	3%	1050	1.727	0.0038	0.955
Kaolin	4,5%	1075	3.510	0.0117	0.836
Kaolin	6%	1099	6.840	0.1485	0.517
Kaolin	8%	1133	14.630	0.0569	0.694
Kaolin	10%	1165	21.340	0.5200	0.469
CMC	1%	1006.7	0	0.0596	0.655
CMC	1,8%	1010.7	0	0.0700	0.800
CMC	2,8%	1016	0	0.1990	0.756
CMC	3,8%	1021	0	0.5720	0.690
Bentonite	3%	1014	1.150	0.0030	1
Bentonite	4,5%	1025	4.300	0.0065	1
Bentonite	6%	1032.8	12.690	0.0060	1

Laminar Flow

From figures 7-14 it can be observed that the Reynolds number used collapses the different concentrations and fluids on the 16/Re line used to describe the flow of water. This is evident for the 75 mm, 150 mm and the 300 mm flumes.

The data that was published by Coussot (3) was used to test the Reynolds number used and it was a good fit. It represents data over a wide range of concentrations, channel slopes and widths. See Figure 5.

The Reynolds number suggested by Kozicki & Tiu (7), which takes the shape factor into consideration was used to check the CMC data and compared favourably with the Reynolds number used in this paper. See Figure 6. The CMC data in Figure 10, using Re Slatter though, is a better fit.

Both the Coussot data set, and the Kozicki & Tiu shape factor formula, confirmed to us that the Reynolds number used for laminar flow is appropriate.

Transition

One may ask what is considered to be transition? Transition was observed where the data deviated from the 16/Re line visually. The same procedure was followed when f.Re/16 was plotted versus Re and the deviation from NAF (normalized adherence function) =1, Slatter (1999), was visually observed. This is obviously subjective, but it was used to try and demonstrate a trend. Figures 3 and 4 depict the data for one concentration, the 8% kaolin in the 150 mm wide flume.

For the Kaolin suspension, the 3% and 4,5 % concentrations behaved more like a Newtonian fluid, with a transition commencing at Re greater than 2100 and with a scattered transition. There was also a vertical jump in the friction factor to the turbulent region as is to be expected for water and as is shown by the work done by Hao & Zhenghai (6). The higher concentrations from 6-10 % followed a smoother transition to turbulence, with the 10% hardly reaching transition. See Figures 7-9 for the Kaolin results.

The same method was used to establish ‘transition’ for all the other data sets.

Figures 10-12 shows the test data for 4 concentrations CMC in the 75 mm, 150 mm and 300 mm flumes. A similar trend as for the Kaolin suspension can be observed.

Figures 13 and 14 shows the data for 3 concentrations Bentonite suspension in the 75 mm and 150 mm flumes. Again a similar trend can be observed as for the Kaolin and the CMC.

Transition occurred over a range of values, and that seemed to depend on the viscous characteristics of the suspension. By this we mean that from the results it can be observed that the higher the concentration, the lower is the value of the Reynolds number at the beginning of transition.

5. CONCLUSIONS and RECOMMENDATIONS

Conclusions.

For laminar flow, the data collapses consistently onto the f=16/Re line, over wide ranges of fluid rheology and flow geometry conditions, provided that the appropriate Reynolds number is used. This can be used for practical design.

From the results presented it can clearly be seen that the transition from laminar to turbulent flow for higher concentration non-Newtonian suspensions is a complex phenomenon and at present cannot be predicted by a single Reynolds number.

The Reynolds number at transition in the tests done varies with concentration and material, as well as for the different size flumes.

The transition for the higher concentration (more viscous) fluids is smoother than for Newtonian fluids. The distinct jump in friction factor at transition, as there is for water, is not present for the more viscous fluids.

The same trend was observed in all three flumes and with kaolin, Bentonite and CMC materials.

For three different materials and different concentrations in three different size flumes and at five different slopes the friction factor in the laminar flow region could be predicted.

Recommendations

The next objective is to try and find a relationship between the Reynolds number at transition and a number of variables that have an influence on transition. These include the fluid and rheological parameters.

Kozicki and Tiu (7) have suggested that the Reynolds number needs to include a shape factor to accommodate the various cross-sectional shapes. This will have to be investigated in more detail. Perhaps there is no simple solution and one may have to use a test flume facility which includes an on line viscometer, to do tests with a given material before designing a large new flume.

REFERENCES

1. CHABBRA, R.P. & RICHARDSON, J.F. (1999) *Non-Newtonian Flow in the Process Industry* Oxford: Butterworth-Heinemann.

2. CHOW, VEN T. (1959). *Open Channel Hydraulics*. New York: McGraw-Hill.

3. COUSSOT, P. (1994). Steady laminar flow of concentrated mud suspensions in open channels. *Journal of hydraulic research*. (32) 4: 535-558

4. DE KEE, D., CHHABRA, R. P., POWLEY, M.B. & ROY, S. (1990). Flow of viscoplastic fluids on an inclined plane: evaluation of yield stress, *Chem. Eng. Comm.* (96): 229-239.

5. HALDENWANG, R. SLATTER, P.T. CHHABRA, R.P. (2000). The flow of Non-Newtonian fluids In Open Channels. *9th International Conference on Transport an Sedimentation of Solid Particles*, Cracow, Poland. 269-280.

6. HAO, Z. & ZHENGHAI, R. (1982). Discussion on Law of Resistance of Hyperconcentration Flow in Open Channel. *Scientia Sinica (Series A)*, Vol XXV no.12

7. KOZICKI, W. & TIU, C. (1967). Non-Newtonian flow through open channels. *The Canadian journal of chemical engineering*, (45): 127-134.

8. SLATTER, P.T. (1994). Transitional and Turbulent flow of non-Newtonian slurries in pipes. Ph.D thesis. University of Cape Town. Cape Town.

9. SLATTER P.T (1999) A New Friction Factor for Yield Stress Fluids. *BHR Group. Hydrotransport 14*. Maastricht Netherlands. 255-265.

10. SLATTER, P.T. & WASP, E.J. (2000) The Laminar Turbulent Transition in Large Pipes.*10th International Conference on Transport and Sedimentation of Solid Particles* , Wraclow, Poland. 389-399.

11. STRAUB, L.G. SILBERMAN, E. and NELSON, H.C. (1958). *Trans. ASCE*, (123): 685-714.

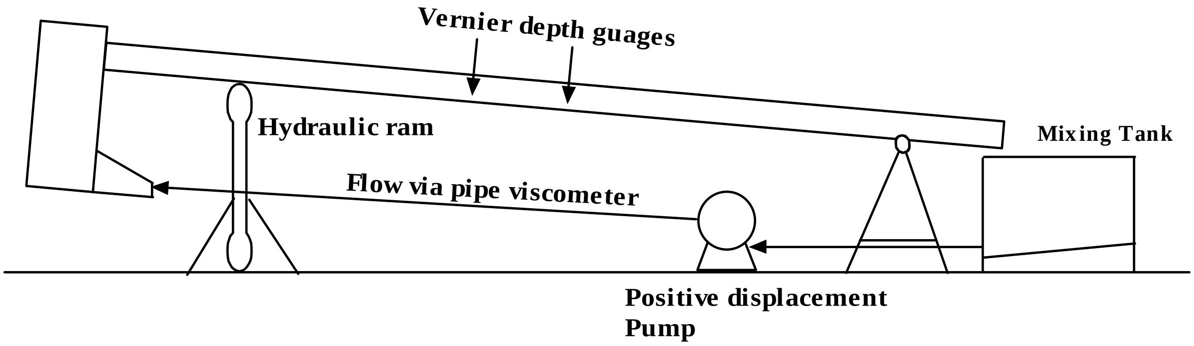

Figure 1. 10 m Flume Rig

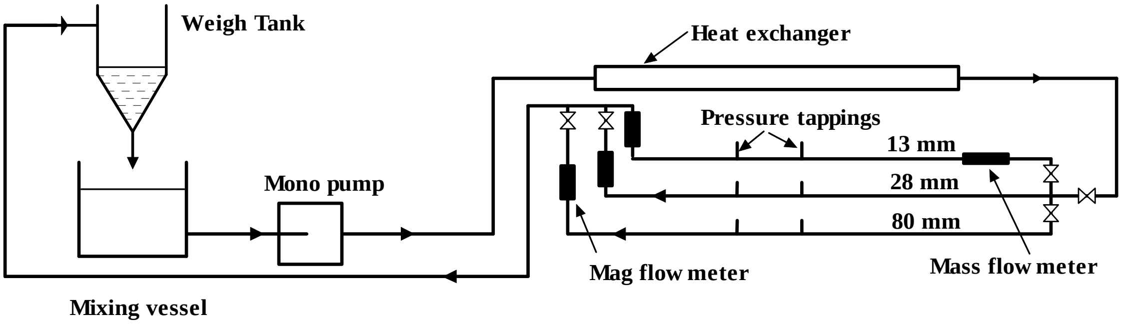

Figure 2. Pipe Viscometer Rig

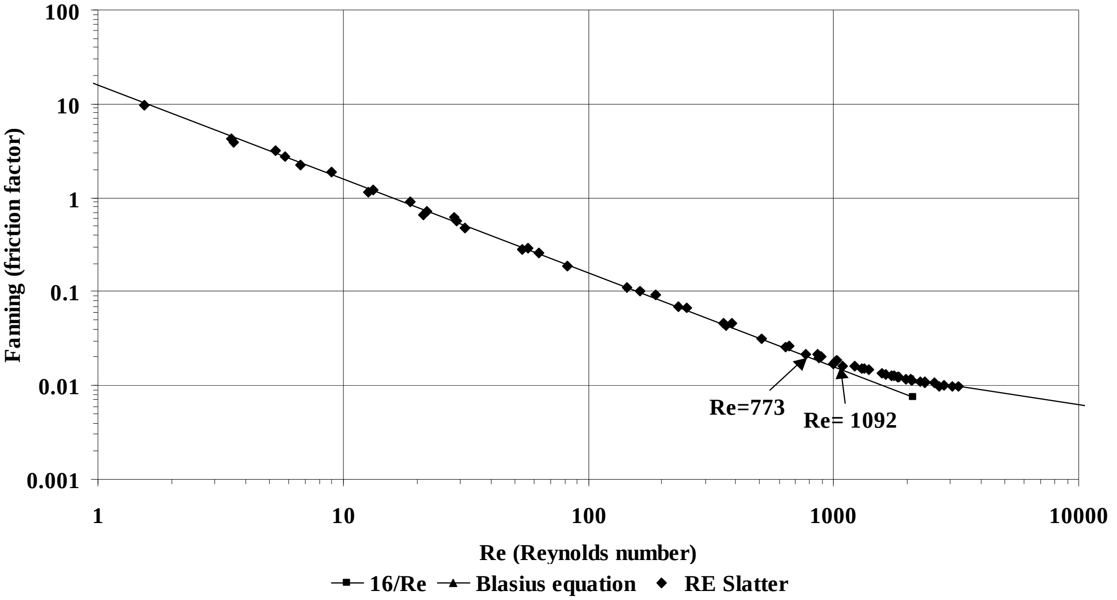

Figure 3 f/Re for 8% Kaolin in 150 mm Flume

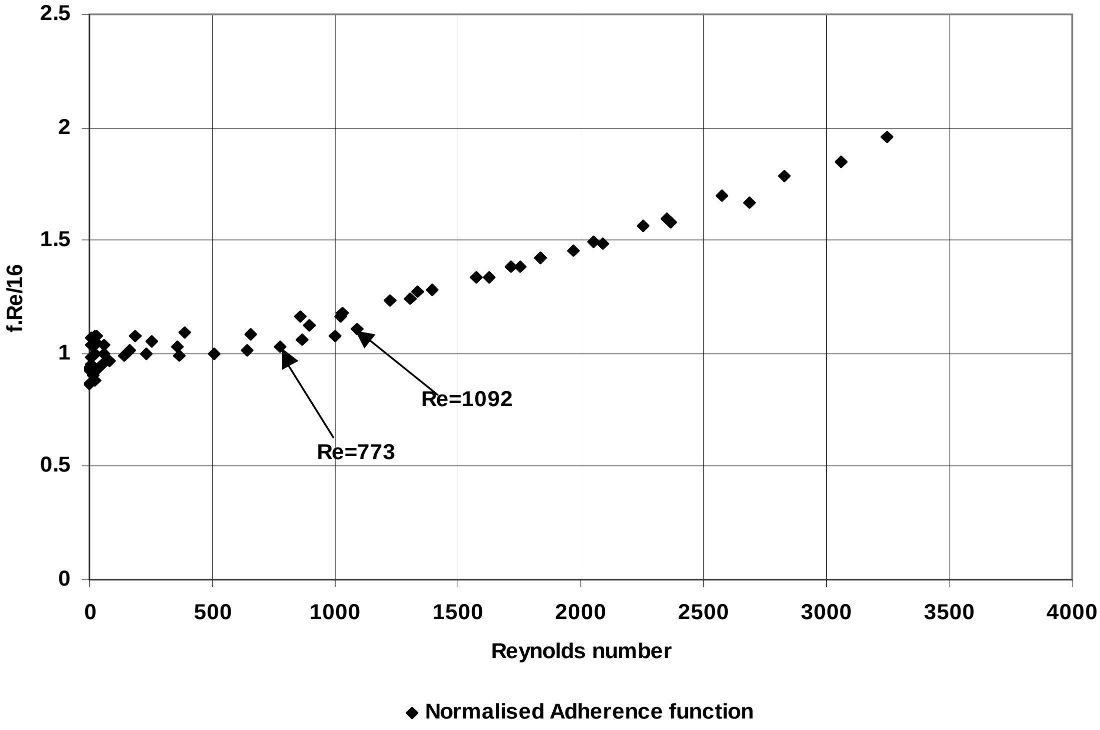

Figure 4. Normal Adherence Function Kaolin 8% in 150 mm Flume

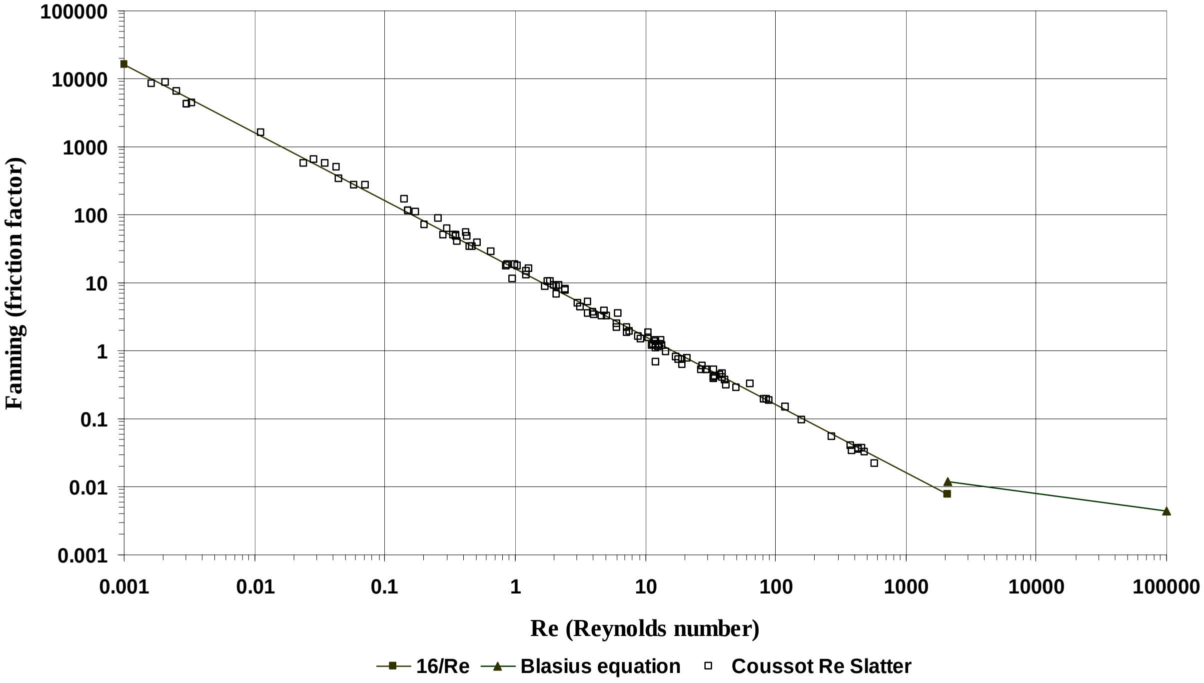

Figure 5. f/Re for clay Suspensions in Flume (Coussot's Data)

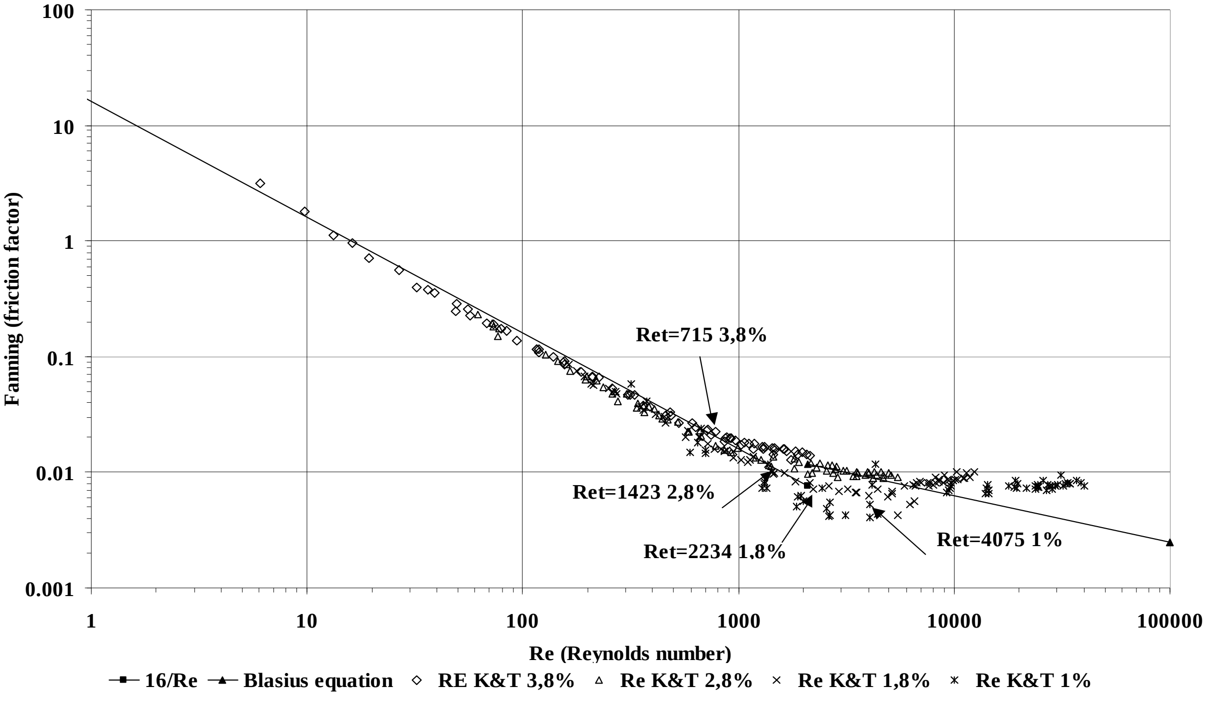

Figure 6. f/Re (Kozicki&Tiu) for CMC 1%,1.8%,2.8%&3.8% in 150 mm Flume

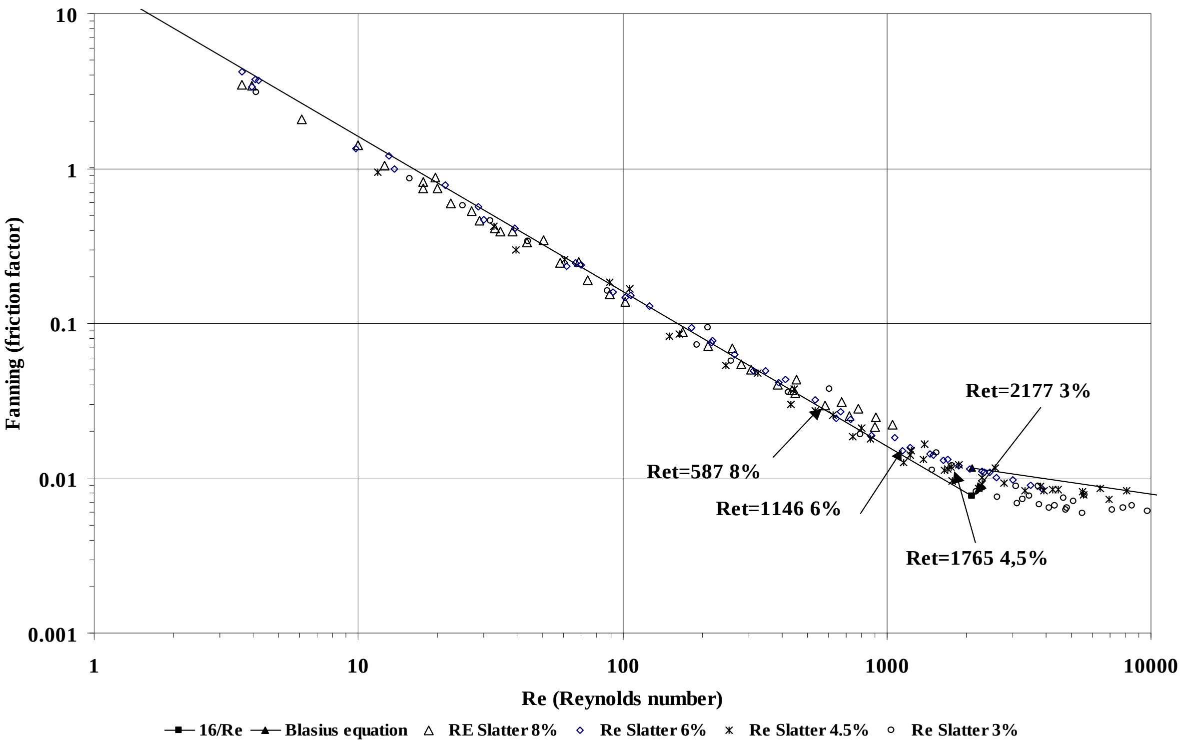

Figure 7. f/Re for Kaolin 3%,4.5%,6%,8%,&10% in 75 mm Flume

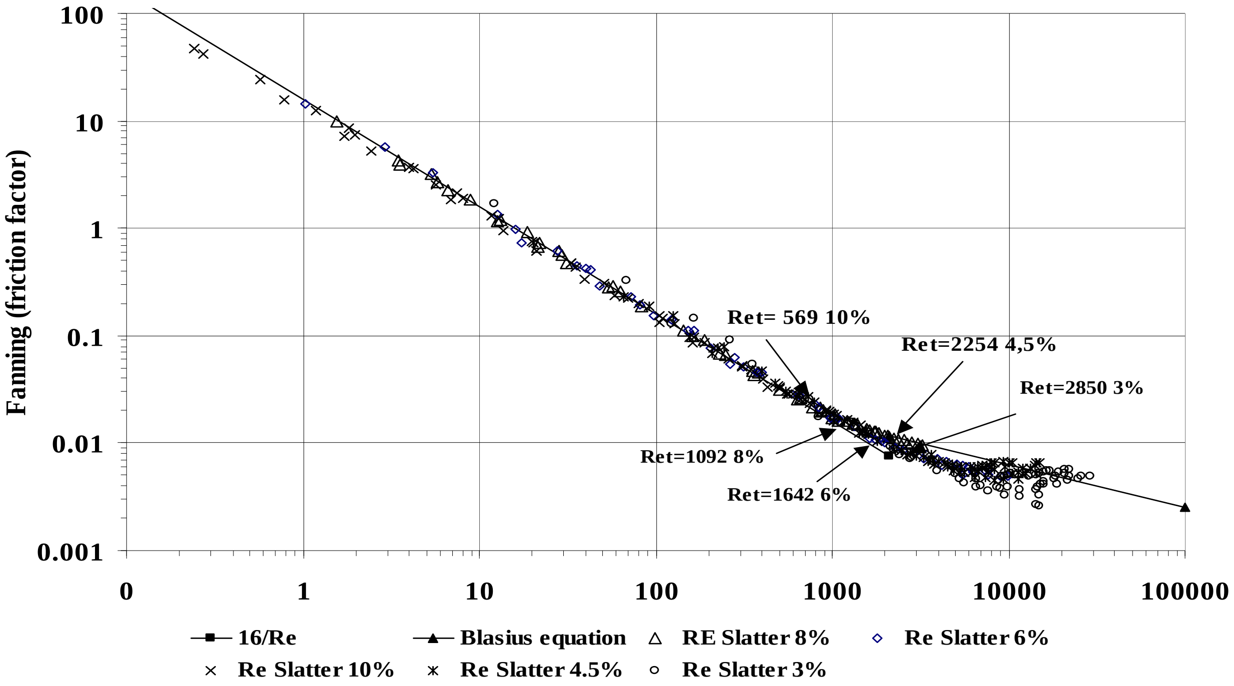

Figure 8. f/Re for Kaolin 3%,4.5%,6%,8%,&10% in 150 mm Flume

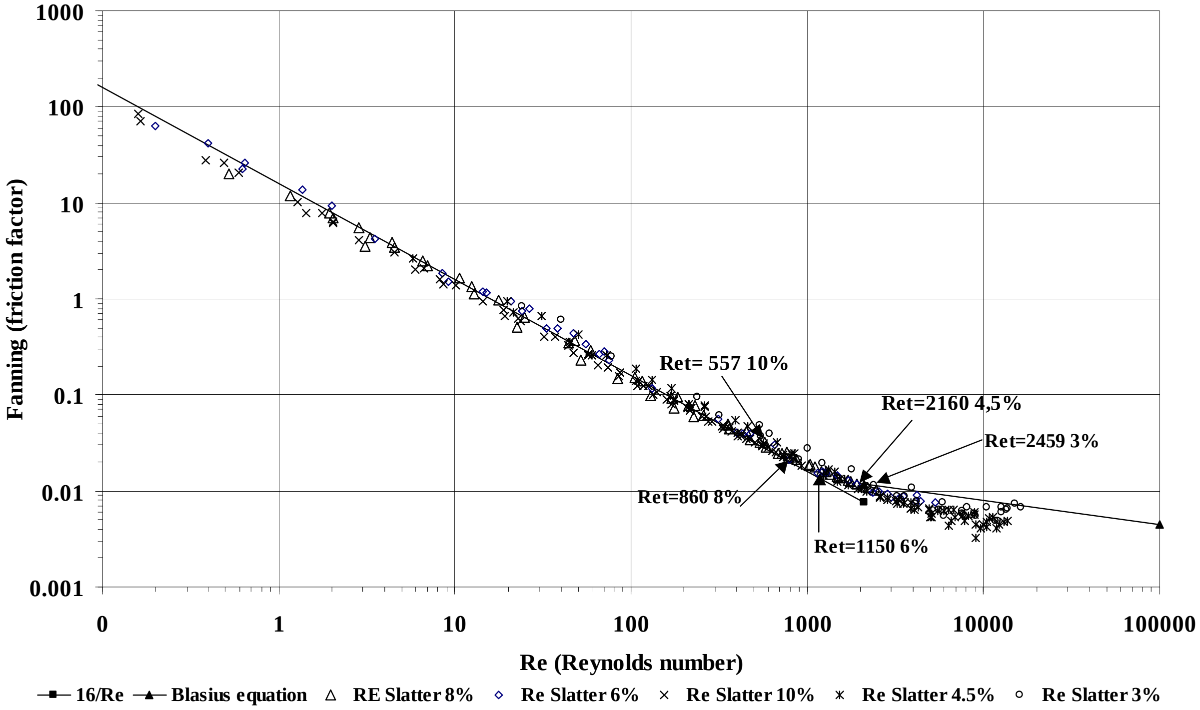

Figure 9. f/Re for Kaolin 3%,4.5%,6%,8%,&10% in 300 mm Flume

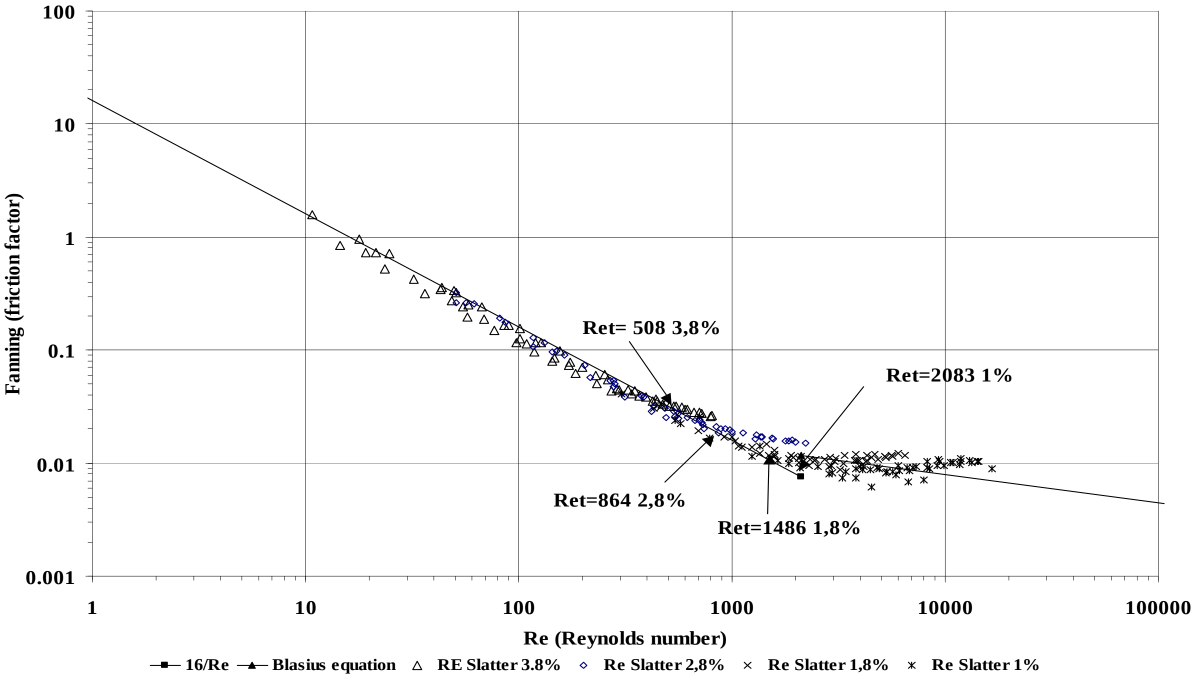

Figure 10. f/Re for CMC 1%,1.8%,2.8%&3.8% in 75 mm Flume

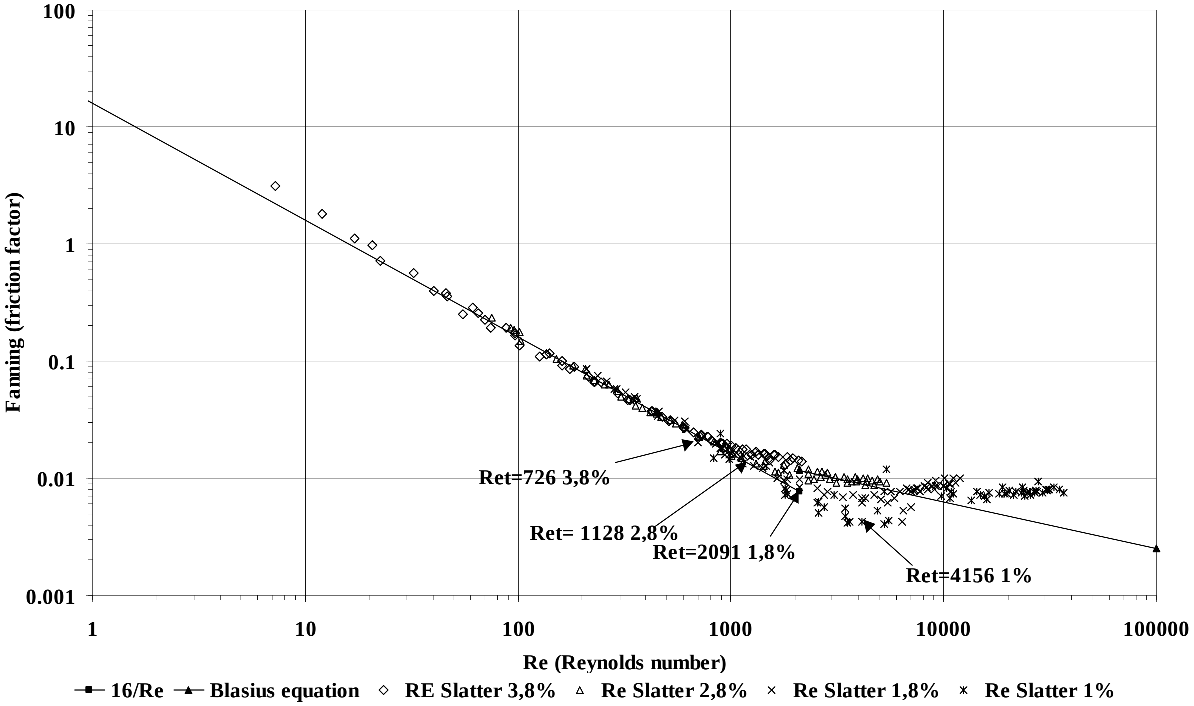

Figure 11. f/Re for CMC 1%,1.8%,2.8%&3.8% in 150 mm Flume

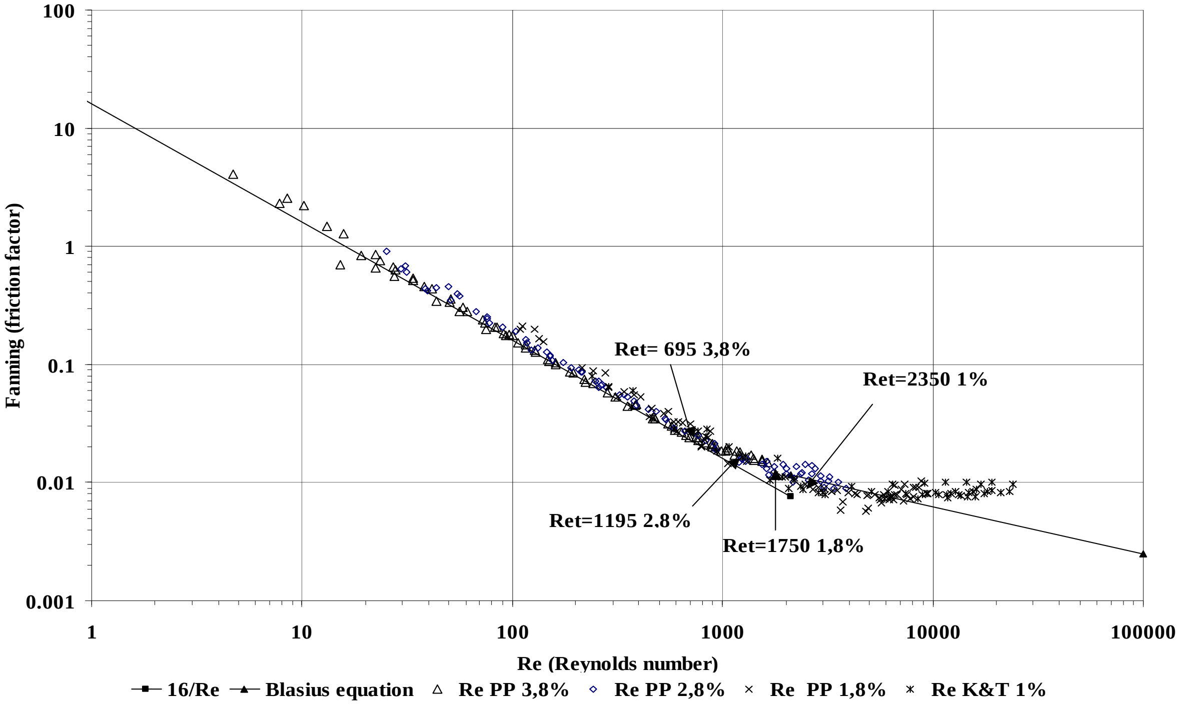

Figure 12. f/Re for CMC 1%,1.8%,2.8%&3.8% in 300 mm Flume

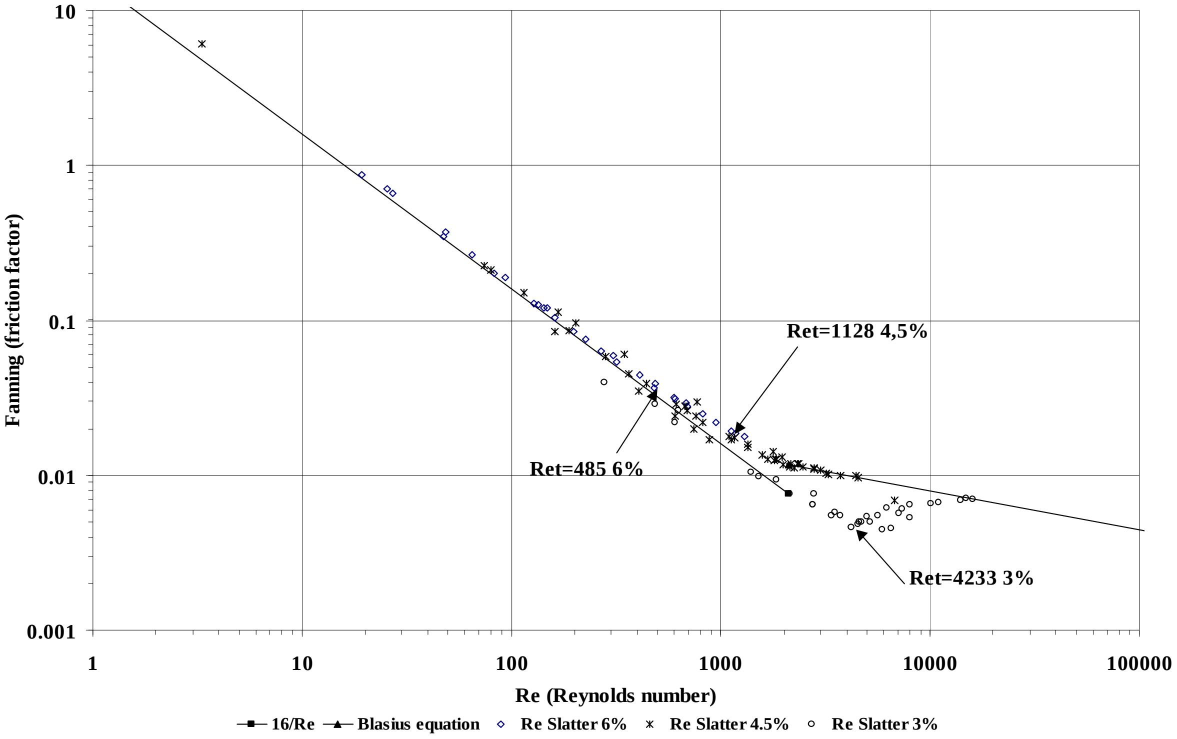

Figure 13. f/Re for Bentonite 3%,4.5%&6% in 75 mm Flume

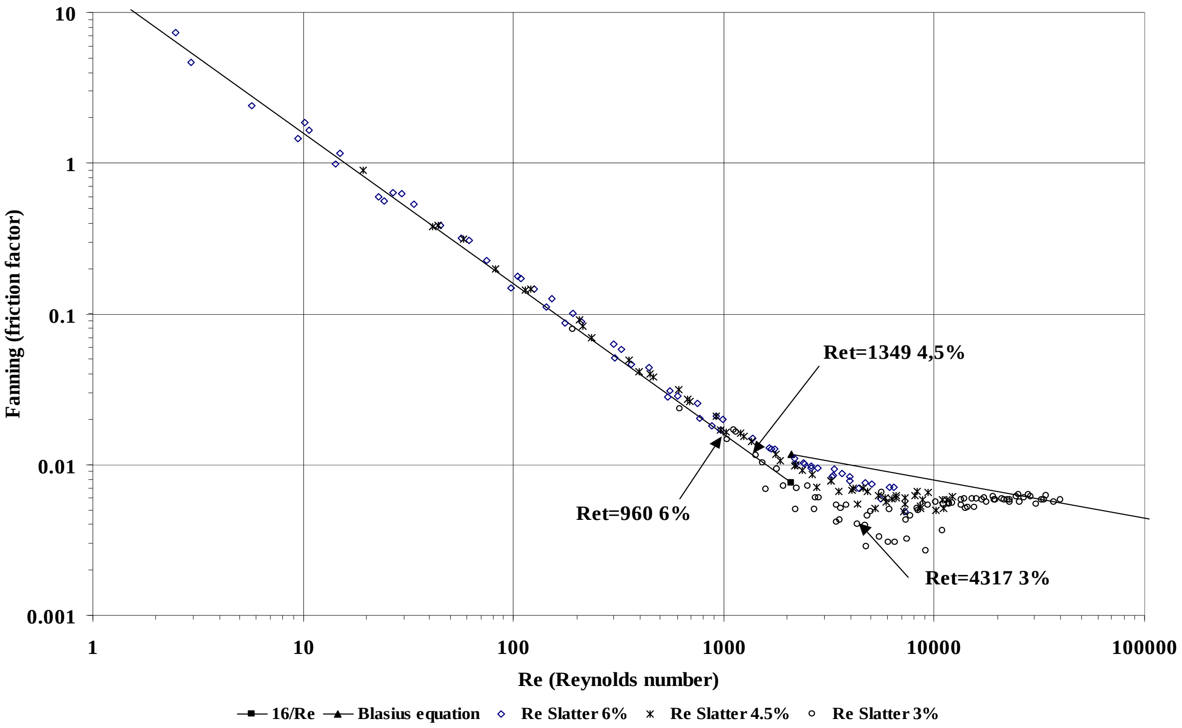

Figure 14. f/Re for Bentonite 3%,4.5%&6% in 150 mm Flume