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Lessons learnt and insights shared: The Diagnostic Mathematics Information for Student Retention and Success (DMISRS) Case Studies Dialogue on teaching first-year mathematics in South Africa

Abstract

The Diagnostic Mathematics Information for Student Retention and Success (DMISRS) project was a collaborative project focused on first-year mathematics, which drew together lecturers in South African higher education institutions and other related organisations. Various themes were explored during the annual

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DMISRS symposia from 2018 to 2022. The project's culmination was a final DMISRS symposium held in November 2023, which focused on the case studies that examine the sub-projects that have emerged over the years. The symposia have tackled critical themes, including collaborative practices and courses that influence graduation rates, the psychological well-being of students, and innovative solutions in first-year mathematics, such as assessments, performance metrics, blended learning approaches and curriculum development. The case studies offer valuable narratives of day-to-day interactions, teaching and learning activities, and shared resources or practices in first-year mathematics education experiences. They showcase successes and delve into challenges and their corresponding solutions, providing practical insights that empower the community of practice across diverse higher education institutions and contexts. As a capstone to the DMISRS project's final symposium, the DMISRS team invited those who contributed case studies to take part in a panel discussion, referred to as the "DMISRS Case Studies Dialogue", on a range of topics, including their own experiences in setting up and implementing innovations or conducting investigations, and their reflections on the impact of and the legacy of the work done as part of this project. This paper captures that conversation as a dialogue among the contributors.

Keywords: *DMISRS, community of practice, mathematics education, first-year experience, teaching and learning*

1. Background and contextualisation

The DMISRS project was conceived as a collaborative initiative to bring together first-year mathematics lecturers from various South African higher education institutions (Prince & Balarin, 2025). The symposia have been a key feature of this project and the most impactful activity of the project as a whole. The project explored many themes through these annual collaborations and symposia from 2018 to 2022 (DMISRS, 2024: 2).

The 2018 themes and central ideas were the following: reflecting on the curriculum, open education resources, staff development and motivation, understanding our students, and best practices in blended learning. The first DMISRS annual symposium was attended by 44 people involved in teaching first-year students and academics from 21 institutions. The first symposium was opened by Prof Mamokgethi Phakeng and set the scene for the project, emphasising the commonality of the problems experienced by mathematics educators at the first-year level. The highlight of this symposium was Prof Jonathan Shock's presentation – "First-year mathematics teaching – the spaghetti approach" – in which he shared his innovative (but practical and implementable) strategies to improve student success (DMISRS, 2024: 2).

During the engagements in 2019, the following themes and ideas were the focus: collaboration, courses that impede graduation, mathematics and/or other disciplines, and the psychological well-being of students. These themes were embedded in the symposium presentations. The second DMISRS symposium was themed "Mathematics and practice", and was attended by 52 academics from 20 universities and two other higher education institutions. The keynote speaker was Prof Lis Lange, who has played a prominent role in driving higher education's teaching and learning enterprise. The highlight of the symposium was the presentation by Prof Suellen Shay, "Courses that impede graduation (CIGs)" (DMISRS, 2024: 3), which addressed critical issues in this context.

In 2020, the focus shifted to themes and ideas related to the following: first-year mathematics solutions, the use of assessments, performance, and blended learning and curriculum development. Due to the COVID-19 pandemic, blended learning gained prominence in 2020. The third DMISRS annual symposium was themed "First-year mathematics solutions: The use of assessments, performance, blended learning and curriculum development during COVID". The symposium was held online and was attended by 56 delegates representing

21 universities. The keynote speaker was Prof Alan Cliff, whose presentation was entitled “COVID-19 impact on teaching and learning and assessment in higher education”. The highlight of the symposium was Mr Noor Ally’s presentation, entitled “Mathematics students – Online learning behaviour and preferences”, in which he expounded on the critical issue of how the pandemic highlighted stark socio-economic disparities between students from disadvantaged and advantaged backgrounds. He emphasised that when transitioning to teaching and learning online, ensuring equitable outcomes for disadvantaged students is more difficult for educators and requires greater intentional effort and entirely new learning resources (DMISRS, 2024: 3).

The engagements in 2021 looked at the following themes and ideas: student profile and mathematics performance, online teaching and assessment. The fourth annual symposium was themed “First-year mathematics: From crisis mode to the new normal” and was also held online. It was attended by 66 participants from 24 institutions. The number of mathematics educators and participants in the DMISRS project was growing, and there was great enthusiasm from all those participating. The keynote speaker was Prof Ahmed Bawa (Universities South Africa (USAf), CEO), whose presentation was entitled “Mathematics for all: Building a social compact”. The highlight of the symposium was the presentation by Dr Anita Campbell *et al.*, entitled “Relating profile to performance in engineering mathematics: A quantitative case study”, in which the authors expounded on the use of the NBT relative importance analysis to inform the revision of the curriculum (DMISRS, 2024: 3).

In 2022, the fifth DMISRS annual symposium was held in a hybrid format, in association with the USAf mathematics community of practice (CoP). The symposium focused on five main themes: teaching first-year mathematics, the future of South African mathematics, critical thinking, support and assessment. It was themed: “What does it take to teach mathematics for and in higher education?”, and was attended by 38 delegates in person and 29 online. The fifth symposium started to emphasise the CoP developing around the DMISRS project, and the inclusion of the USAf mathematics CoP obviously assisted it. The keynote speakers were Prof Loyiso Nongxa, whose presentation was entitled “South African mathematical sciences in 2040”, and Prof Alfred Msomi (MUT), whose presentation was entitled “Imperatives on the integration of technology in teaching and learning at an ICT-challenged university in South Africa: Ontology of a mixed methods approach”. The highlights were the presentations by Mr Darlington Hove (MUT) – “Reflections on support and assessment of engineering mathematics during COVID-19” – and Mr Neil Eddy (Ubuntu Mathematics Institute) – “What have we not tried in our pursuit of mathematical success?” A noteworthy aspect was the trend for many symposium presentations to embed the use of diagnostic data in the evaluation of curricula and strategies for improving student success – a clear indication of the impact of the DMISRS project (DMISRS, 2024: 3). A key takeaway from this penultimate symposium was the value of belonging to and engaging in activities as a community of practice that allow for sharing of knowledge, insights and experiences. This inspired the theme of the symposium in the following year and underscored the need for reflection on both the individual and shared journeys throughout the project.

2. Capturing the progress of a CoP: Lessons learnt and insights gained

The final DMISRS symposium, held in 2023 and attended by 46 delegates (26 online delegates and 20 in-person delegates) representing 17 universities, tied together the foci of previous engagements and turned to the following: bridging the gap, students’ success initiatives in

institutions and the design of student support solutions. The sixth annual symposium was hybrid and parallel with the DELTA 2023 mathematics conference.¹ It seemed fitting that the DMISRS project should end at the same time the international DELTA conference was held. The theme of this symposium was “Lessons learnt: The DMISRS case studies in teaching first-year mathematics in South Africa”. The delegates explored and shared best practices in teaching first-year mathematics to exchange valuable insights and lessons learnt at this critical stage of students’ higher education journey (DMISRS, 2024: 3). The symposium was followed by a closed panel discussion (comprising those who contributed case studies to the DMISRS project), referred to as the “DMISRS Case Study Dialogue”.

This paper shares the insights, outcomes and recommendations elicited through a semi-structured discussion facilitated by members of the DMISRS project team. The main objective of this exercise was to explore questions central to the work done within a CoP – exploring common ground, sharing experiences and resources, collectively developing solutions to challenges in the present and laying the groundwork for tackling those hurdles that have yet to present themselves. This objective frames the research in the context of this paper as a way of examining the project activities through the lens of CoP theory.

Research question: How do the activities that formed part of this inter-institutional collaborative project contribute to the shared practice of the CoP?

Our methodology comprised two complementary phases: first, structured questioning of individual case study contributors to capture specific institutional experiences; second, facilitated group reflection to articulate common areas of practice and shared learning across the DMISRS community.

2.1 A community of practice in conversation

Ahead of the 2023 symposium, DMISRS participants were invited to contribute their case studies, offering practical examples of teaching and learning innovations in their own context, reflecting on typical daily interactions with students, sharing resources and practices, and sharing narratives about the experiences and challenges (and possible solutions) associated with teaching mathematics at the first-year level. Contributors were encouraged to reflect on successes and explore the challenges that arise in this teaching context, providing an opportunity to understand what did not work and offering their views on the lessons learnt. The intention was to allow these reflections to form the background for future professional development work (DMISRS, 2023a).

Underpinning the preparation for the 2023 symposium was the understanding that a CoP has certain characteristics that set it apart from other groupings (Thompson, 2005: 151). Wenger (2011: 1) provides a useful definition of CoPs:

Communities of practice are groups of people who share a concern or a passion for something they do and learn how to do it better as they interact regularly.

These CoPs have three elements that must be present in order for us to call a group a CoP – a domain, a community and a practice – and they must also have a set of typical activities in which they engage, such as collaboratively tackling key problems and identifying gaps, seeking knowledge and sharing experiences, reusing assets and strategies, and coordinating efforts as a community with a shared focus (Wenger, 2011: 2, 3). These elements provided

1 <https://uctcmc.eventsair.com/delta-maths-conference-2023/>

a framework for us to plan the panel discussion in a way that would allow us to explore the learnings and experiences of the panel of DMISRS contributors and also gain some insights into how its participants perceive this CoP and its activities during the DMISRS project and the directions that it may take in the future. In the context of DMISRS, these three elements manifest as follows: the domain encompasses first-year mathematics education; the community consists of educators from across the higher education sector; and the practice is evidenced through the case studies that surface the collaborative work of this CoP in action.

2.2 Contributors and their case studies

Most contributors presented their case studies at the sixth DMISRS symposium (2023). Table 1 lists presenters and their case studies from this engagement.

Table 1: Contributors and case studies presented at the sixth DMISRS symposium (2023)

Contributor: Liezle Boshoff (LB) – Cape Peninsula University of Technology (CPUT) Case study: Designing an activity to support the development of critical thinking (Appendix A) A reflection on the design and implementation of an activity to support the development of critical thinking, creativity and certain graduate attributes in a first-year mathematics service course for engineering students.
Contributor: Anita Campbell (AC) – University of Cape Town (UCT) Case study: How Gradescope revolutionised my marking process (Appendix B) An educator's insights into the value of a new marking tool and a new approach to marking in their context. This approach has made marking more consistent, flexible, transparent and faster for the lecturer. The students have a tool that helps to collect feedback on their work and keeps track of their progress.
Contributors: Ekaterina Rzyankina (ER) and Frikkie George (FG) – Cape Peninsula University of Technology (CPUT) Case study: Exploring treescapes concept for teaching mathematics: Ethnomathematics perspective (Appendix C) An innovative educational approach: integrating the concept of "treescapes" – the visual and spatial arrangement of trees within a landscape, encompassing their composition, structure and aesthetic qualities – into mathematics education.
Contributor: Vasuthavan Govender (VG) – Nelson Mandela University (NMU) Case study: Traversing the gap between Grade 12 mathematics and first-year mathematics: My first-year personal reflections (Appendix D) A set of insights and observations from an educator based on their experiences in supporting students transitioning from Grade 12 mathematics to first-year mathematics, as well as those of other teachers and lecturers working at the coalface.
Contributor: Darlington Hove (DH) – Mangosuthu University of Technology (MUT) Case study: Reflections on support and assessment of engineering mathematics during COVID-19 (Appendix E) An exploration of the challenges and successes of the support and assessment mechanisms developed and implemented for mathematics students studying engineering during and after the COVID-19 pandemic.

Contributor: Syamala Krishnannair (SK) – University of Zululand (UNIZULU)

Case study: Summary of the activities that the University of Zululand has done with the DMISRS project (Appendix F)

A look at introducing new teaching practices and assessment strategies at the University of Zululand as part of its participation in the DMISRS project.

Contributor: Narenee Mewalal (NM) – Durban University of Technology (DUT)

“It is hard work keeping the attention of a distracted audience” (Appendix G)

An investigation into the challenges and successes of a project where several teaching strategies were implemented to foster interest in the subject (mathematics) and improve success rates. The outcome significantly improved students’ positive self-concept and pass rates.

Contributor: Mashudu Mokhithi (MM) – University of Cape Town (UCT)

Case study: Enhancing mathematical thinking and metacognitive skills in a first-year mathematics course (Appendix H)

An overview of the insights and observations – from both the student and the facilitator perspectives – on a project that introduced mathematical thinking workshops as an additional support intervention to augment the student learning experience and enhance students’ mathematical thinking and metacognitive skills in first-year mathematics courses.

Contributor: Pragashni Padayachee (PP) – University of Cape Town (UCT)

Case study: A community of practice teaching and learning mathematics: Collaboration, collegiality and collective reflection (Appendix I)

Reflections on establishing a community of practice for teaching and learning mathematics at the first-year level and its value in teaching mathematics in the South African higher education sector.

Contributor: Tatiana Sango (TS) – University of Cape Town (UCT)

Case study: Blending the equation: The final year of the DMISRS project (Appendix J)

Insights from a blended learning project under DMISRS aimed to address the difficulties that students face when transitioning from high school to university-level mathematics courses through the design and pilot implementation of an adaptive learning tool.

Contributor: Jonathan Shock (JS) – University of Cape Town (UCT)

Case study: The spaghetti approach to teaching mathematics (Appendix K)

A reflection on several small interventions implemented to try to engage with students at different points on their journey through studying mathematics, from supporting those who struggle to engaging those who find the pace too slow.

Contributors: Sanet Steyn (SS) and Tatiana Sango (TS) – University of Cape Town (UCT)

Case study: Beyond the numbers: Exploring the role of academic literacy in mathematics performance (Appendix L)

An investigation into the underlying relationship between performance in mathematics and academic literacy tests among prospective first-year university students.

The closed panel discussion session (facilitated by Sanet Steyn) at the end of the symposium was structured around a set of predetermined questions and also picked up on the themes and discussion points that emerged during the case study presentations in the morning sessions. To allow the DMISRS team to capture the conversation, and in addition to recording the session, an interactive survey on Wooclap² was open throughout the session, for both

² <https://www.wooclap.com/>

those in attendance at the symposium venue and the participants who joined online (DMISRS, 2023b). The survey was also kept open and available to participants to review their responses for a week after the panel session. The remainder of this paper will present the conversation captured using these different tools, starting with introductions, then moving on to the thematically grouped questions posed to the panel and to some key takeaways from this discussion, and ending with final concluding remarks.

3. Connecting the dots and sharing thoughts: The DMISRS Case Study Dialogue

3.1 Exploring the common ground

Throughout this project, the DMISRS symposia have allowed the community to explore various themes and unpack several challenges associated with teaching mathematics at the first-year level in the university context.

In addition to the topics chosen as themes for the symposia, several subjects emerged from the interactions during these events, and certain topics became more prominent as the project continued. Referring again to Wenger's (2011: 4) conceptualisation of a CoP, one aim of this grouping is to collectively take responsibility for exploring the domain in which they practise, and to identify the gaps and issues that need to be addressed. With that in mind, the starting point of our panel discussion was to capture reflections on the overarching themes that the contributors could see emerging from the discussion of their case studies or to which they were drawn when they started working on their individual initiatives. (See an extract from the conversation around Question 1 in Figure 1 below.)

Question 1: This last symposium mainly focused on reporting on the lessons learnt during the DMISRS project through exploring the case studies of some of the projects that members have embarked on since the project's inception. What would you say are the overarching themes that came to mind as you were listening to (and preparing for) the case study presentations?

Using technology to teach, learn, assess and research mathematics	1st year Mathematics CoP, student support, Blended learning	Ethnomathematics in SA context - use treescape as a teaching concept	Mathematics as we teach seem to be more procedural at first year and that's not enough for our students success.	Community; Diversity; Experimentation; Development through Maths.
The Key themes remain: the transition from HS to Tertiary; and the development of mathematical thinking	The state of mathematics in South Africa.	Enhancing first-year mathematics education through reflection and innovative teaching methods.	The difficulty in transitioning from high school to university. The first-year experience.	Rethink how we teach mathematics to include aspects like mathematical thinking and critical thinking, because its not sufficient to keep teaching as we do.
Integrating the use of technology in the teaching, learning and assessment of mathematics	The importance of a COP of mathematics university teachers	Modules that impede graduation and psychological well-being of first-year students	Transition, critical thinking, problem solving, formal mathematics including proofs: difficulties students have; the need for an ongoing CoP	
The key theme that came to mind is interventions in the aftermath of the Covid-19 disruption. The Covid-19 pandemic presented new challenges further down the supply chain but brought new opportunities in terms of the acceptance of online learning. However, the digital divide still exists or possibly widened.				

Figure 1: Snapshot of the responses to Question 1

The main comments captured in Figure 1 reflect the overall feeling about the transition from high school to university mathematics, the need to move beyond procedural teaching to foster mathematical and critical thinking, and the role of technology in enhancing teaching, learning and assessment. The themes the participants reflected on in their responses to Question 1 indicate the common challenges across the sector. However, as evident from the case studies, practitioners' responses to challenges in teaching and learning mathematics at the first-year level varied in focus area and approach despite these commonalities. Question 2 was posed to gain a better understanding of the motivation behind the initiatives that each of the contributors started.

Question 2: Many of these themes are inextricably linked to the core challenges we face in the higher education sector, specifically in mathematics education in this context. Can you share more about the impetus for change in individual contexts and why you chose to embark on the initiative or investigation you shared in your case study?

Seeing that things weren't working and students were struggling	Seeing that students struggled with mental health issues	Noticing a massive gap between high school maths and university maths	Wanting to give a voice to those who don't want to speak up in class
The courses impeding graduation project	Mathematics in general is abstract (students need help with visualising concepts)	Seeing the lack of mentoring/support for first year students.	Moving the teaching and learning from classroom to the outside environment.
To build students confidence, let them show what they know rather than what they do not know	Change the role of Maths in Engineering	Increase the awareness of a COP	Enhance the first-year experience
People should apply maths, not just learn it.	The differing circumstances our students find themselves in cannot be ignored	The reasons for poor performances/student apathy in respect of mathematics at my institution	My 30 year career in mathematics education

Figure 2: Snapshot of the responses to Question 2 – Impetus for change

Question 2 encapsulated two main ideas: the impetus for change and the reason for embarking on a particular initiative. From the responses (summarised in Figure 2), participants saw the impetus for change in recognising that traditional approaches to teaching mathematics are not meeting students' needs, particularly in their transition from high school to university. Concerns included students' struggles with mathematical abstraction, a lack of mentoring and support, mental health challenges, and the need to build confidence by focusing on their strengths. Additionally, there is a motivation to make mathematics more applicable, foster a sense of community, and rethink the role of mathematics in disciplines like engineering to enhance student engagement and success.

Wanted to experiment with lots of different interventions/implementations	Trying to understand what was needed to bridge the gap (school-university) in mathematical thinking
The use of GeoGebra brings an element of visualisation in the student mind and get to understand the concept better.	Create different teaching and learning environment.
Adaptive learning has the potential to create a unique starting point for first-year students.	Embracing ICTs in education has the potential to revolutionise the learning and teaching process, creating dynamic and engaging learning environments for students.
We came to realisation how important treescape concept in mathematics is; and valuable to respond to UN Sustainable Goals of Development.	Invite colleagues to see value of COP - to form their own or join in the community
The effectiveness of interventions was largely dependent on the backgrounds of the students.	The topic was a result of my attempt to explore the reasons for poor performances/student apathy in respect of mathematics at my institution.

Figure 3: Snapshot of the responses to Question 2 – Reason

The second part of Question 2, the reason for a particular initiative, is captured in Figure 3. Although each of these contributors had a particular focus and potential solution in mind, a common thread throughout these comments suggests a central concern around understanding and addressing the obstacles that first-year students face when transitioning from studying mathematics in high school to studying mathematics in a higher education context.

3.2 A sometimes-bumpy road

Addressing the gap between secondary school and university has been the topic of several studies (see studies such as Dlamini *et al.*, 2020; Ferguson *et al.*, 2019; Govender, 2020; Motsabi *et al.*, 2020; Van der Merwe *et al.*, 2020) and the persistent focus on this issue highlights the complexity of finding and implementing effective and scalable solutions. Given the differing institutional contexts, resource availability, student and staff commitment to these proposed solutions, as described in the case studies (see Appendices A to L), sharing, documenting and analysing the implementation hurdles is valuable to a broader community. Figure 4 shows the panel's discussion of the challenges they faced (and may continue to face) when implementing the initiatives they have been working on.

Question 3: You talked a bit about your motivation or rationale for the topics you chose to investigate and hinted at some of the demands associated with each. Were there any particular challenges you faced – expected or unexpected – when you embarked on your undertaking? Do you foresee those same challenges continuing to impact your efforts?

Students being overwhelmed with options.	Lack/shortage of resources.	Resistance from colleagues.	Students not having time to take advantage of resources provided.	Not enough space to cater for all students.
Students hacking into Gradescope.	Student attendance in class.	Unequal student access to technology.	In service training the teachers.	Buy-in of education leaders and traditional stakeholders.
Deciding what to leave out of the Maths syllabus.	Assessing critical thinking activities.	Buy in from a wider range of practitioners and trainers.	Restrictive and over-crowded curriculum.	Integrating AI strategically into my students learning of mathematics.
Finding ways to help students who get stuck without telling them what to do.	It is time-consuming to design activities that encourage critical thinking.	How to incorporate ethnomathematics as part of the lesson?	Convincing engineering students to see the value of learning mathematics.	Not enough time in lessons to do things the way you believe is the right way.
Lack of ongoing participation. Students found novel interventions great, but they quickly got tired of them.	My insufficient ICT skills plus that if students to design teaching tools.	Expected - motivating students to participate in what we think might benefit their learning.	Students thinking that the workshops are for "slow" students.	Some types of development (such as improvement of students' critical thinking ability) are very difficult to assess.
Poor student readiness for higher learning, socio-economical differences, very little exposure to technology.	Students' technology skills. There is a need of training for students to work successfully in the technology environment.	Structural issues - students in one class may be doing different degrees, and thus the constraints on them may be very different.	Time- I want to teach mathematics in an integrated way showing applications from various engineering disciplines.	In my context, there is poor wifi connectivity and students no longer receive data from the university. Added to that, the vast number of students come to university from poorly equipped high schools.

Figure 4: Snapshot of the responses to Question 3

These hurdles (captured in Figure 4) range from factors beyond educators' control, such as the socio-economic conditions that shape a student's direct environment, to challenges that can be addressed through collaborative work in the education sector, and even aspects that smaller, targeted interventions can directly influence. Working as a collective in a CoP offers distinct advantages in navigating these complexities. As Wenger (2011: 2) points out, a CoP is not just a group with common interests. However, members are practitioners who actively engage in developing resources that can address their common challenges and may benefit the group as a whole. Moreover, a CoP endeavours to understand its domain thoroughly enough to allow this community to create the most appropriate resources and initiatives to do this work (Wenger, 2011: 1).

For the CoP represented by the members of this panel, several factors may hinder their ability to continue with their initiatives or take their work forward. Among the obstacles that were raised (see Figure 5 for an extract of the responses to Question 4), two key issues stand out:

1. Resourcing and capacity – some of the projects require funding that will enable the development or acquisition of the necessary tools, software or materials, or the employment of human resources;
2. Collaborative work – within institutions and between both local and foreign institutions – is crucial, but requires commitment and an enabling environment to succeed.

Question 4: Speaking of challenges and overcoming them, are there any things you would say are on your wish list that could help you address them or overcome some of the obstacles you might face in implementing your plans going forward?

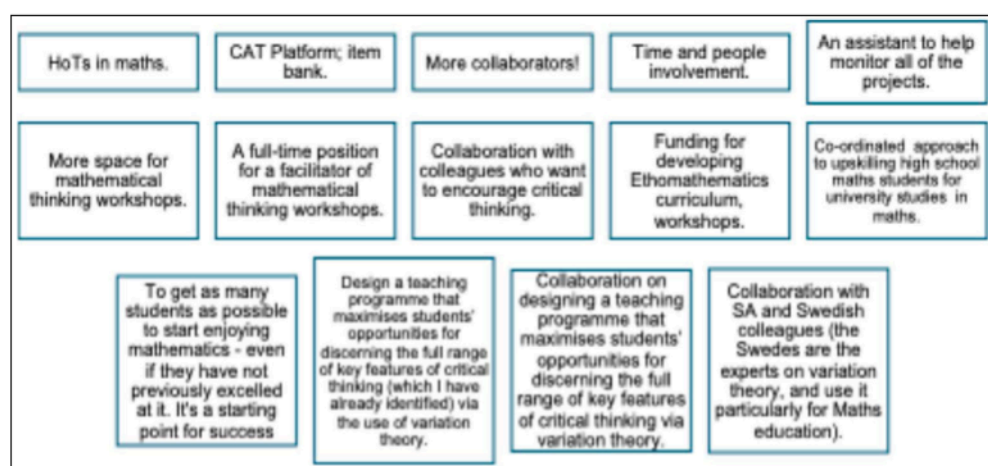


Figure 5: Snapshot of the responses to Question 4

3.3 Looking towards the future

As mentioned earlier in this paper, one goal of the sixth DMISRS symposium – and, by extension, the panel discussion – was to conclude this project with a review of the lessons that were learnt throughout the project, and also to get a sense of what the future might hold for the initiatives that were started during the project's six-year run. Figure 6 provides an overview of the panel's responses to Question 5 regarding the way forward for their sub-projects and studies.

Question 5: Considering the challenges you mentioned, the things you would need, and the observations you made during this process, where would you say you need to go next? Are you going to continue using the solutions that you have presented here or refine them further? Or is it back to the drawing board or on to newer and better things?

Involve postgrad students to do research with me.	Less theory, and more thinking/application in Maths.	The mathematical thinking workshops will continue.	I will keep throwing Spaghetti at the wall to see what sticks.	I will continue to look at more collaborators.
I will make a video demo on using Gradescope.	More workshops, improved based on lessons drawn from the pilot project.	We want to create an open access resource about how to run mathematical thinking workshops.	Longitudinal studies on the long-term effects of the mathematical thinking workshops.	Engage with school educators through workshops for the purposes of research contribution.
Targeting getting calculus-ready students entering tertiary mathematics education.	Integrate learnings/lessons from other case studies into our work.	Add modelling to curriculum, take unnecessary Maths in Engineering curriculum out.	Creating a Community of Practice for best lecturers and teachers.	I hope to do design-based research - continue and refine at the same time.
Reflecting on the role of the CoP and its participation in mathematics education nationally.	Writing a paper and workshoping the ideas of the construction of the domain of real numbers.	Seeing how we can support those on the mathematical thinking workshops going forward.	We have applied for UCDG funding for lecturer and tutor training to expand the CoP around mathematical thinking workshops.	Redefine the cop to include subgroups assigned to particular areas in maths ed. to inform curriculum and its interpretation at schools, teacher education and first-year courses.

Figure 6: Snapshot of the responses to Question 5

By and large, the contributors had specific ideas around how they would like to take forward the work presented in their case studies. In some cases, this included efforts to institutionalise the initiatives piloted in these contexts. In others, the plans included additional piloting and even collaborative efforts to implement potential solutions across different institutions.

In addition to continuing with the work started in these subprojects, continuing capacity-building, enhancing practices and creating shared resources were identified as important next steps for this CoP. Based on Wenger's (2011: 1, 2) ideas around domain, community and practice, being a member of a CoP implies: (1) a responsibility to the domain and continuous professional development to attain a shared level of competence that distinguishes its members from others; (2) seeking opportunities to participate in joint activities and sharing knowledge and experiences; and (3) that members are practitioners and must use their expertise to advance the community and the domain. Figures 7, 8 and 9 provide an overview of the panel's ideas about professional development and future collaborations within the CoP in response to Questions 6 and 7, respectively.

Question 6: Just within this collection of case studies and from what you have shared about your experience thus far, it is safe to say that there are some commonalities between the successes and challenges you all encountered in your projects. How could the lessons you have learnt or the solutions you have designed be useful for professional development within your own context or beyond?

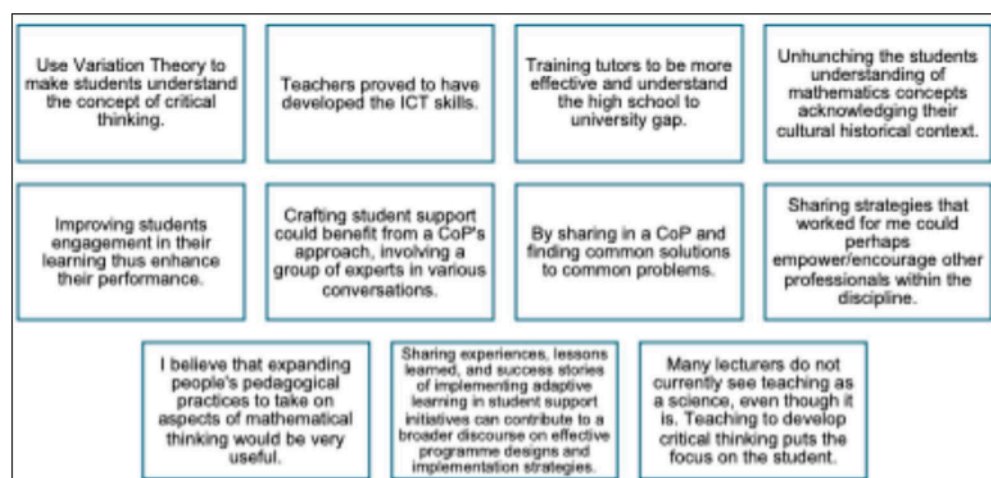


Figure 7: Snapshot of the responses to Question 6

The panel highlighted several topics (see Table 2) that they think the CoP should include as future areas of interest, including developing and sharing strategies to facilitate the development of critical thinking skills in the context of the mathematics classroom and exploring novel ways to unpack students' understanding and application of mathematics in the STEM disciplines.

Tables 2 and 3 below summarise the panel's responses to Question 7. Table 2 provides an overview of the comments regarding the potential for project collaboration. Table 3 is a matrix that maps the common interest areas where participants indicated links between different projects described in the case studies.

Table 2: Summary of comments on links and potential for collaboration between projects (Question 7)

Question 7: After hearing more about the other case studies, where do you see your particular example/solution could be useful to one of the colleagues working in a different context? Or where can you see any of those other solutions working within your context/solving your problem?	
Project	Comments
1) Padayachee (UCT): CoP for teaching and learning of mathematics	Potential touch points between what is planned for the CoP and other projects: - TS: Wider CoP participation; mathematical thinking for NBT MAT; Grade 12 gap (through CAT in Grade 11) - JS, MM and LB: Development of critical thinking ability in or through mathematics - LB: What is important to learn in, for example, engineering maths, and how should it be taught? - FG: Developing a focus (or a subgroup) for the CoP in the SA context on using African theories (like the treescape/ethnomathematics project)

Question 7: After hearing more about the other case studies, where do you see your particular example/solution could be useful to one of the colleagues working in a different context? Or where can you see any of those other solutions working within your context/solving your problem?	
2) Krishnannair (UNIZULU): DMISRS academic support at the University of Zululand	TS: We are planning a collaborative project with UNIZULU. We want to implement ALEKS with Calculus 1 students.
3) Mokhithi (UCT): Mathematical thinking and metacognitive skills	Some other contributors who see potential for some collaborative projects: - LB: The concurrent development of critical and mathematical thinking skills - TS: Mathematical thinking in NBT MAT
4) Boshoff (CPUT): Development of critical thinking	In addition to the potential collaboration with other colleagues working on mathematical thinking (MM and JS on mathematical workshops; TS on mathematical thinking for NBT MAT), LB mentioned a possible link between this project and another project where they might explore the role of critical thinking in mathematics Olympiads
5) Govender (NMU): Traversing the gap between Grade 12 and first year	There is potential for collaboration: - JS/MM: Mathematical thinking workshops and a focus on this gap - TS: Addressing the Grade 12 gap through CAT in Grade 11
6) Shock (UCT): Spaghetti approach/ curriculum reform	TS noted possible links with three subprojects related to her project: Bootcamp (virtual) and potentially a link to Grade 11 project / explore virtual implementation of the maths learning centre and possible replication at the school level The idea of “bonus marks”, one of the possibilities that was mentioned in relation to the “spaghetti approach”, could be promoted with the implementation of the NBT Aleks solution
7) Sango (UCT): Blending the equation	JS saw potential for including some form of implementation of this project in the “spaghetti approach” mentioned in this discussion
8) DMISRS CoP: Assessment-related ideas/issues and strategies for the sector (and further engagement in the CoP)	There was a consensus among the group that one theme that the CoP may need to focus on in the near future is assessment and mathematics education (specifically supported by PP/LB/AC/JS/MM)
Projects with general appeal that might be of interest to the broader community: - Mewalal (DUT): “It’s hard work keeping the attention of a distracted audience” - Campbell (UCT): “How Gradescope revolutionised my marking process”	

Table 3: Case studies: Matrix of common interests (Question 7)

	PP	MM	LB	JS	TS	AC	NM	FG
1) Padayachee (UCT): CoP for teaching and learning of mathematics		*	*	*	*		*	*
2) Krishnannair (UNIZULU): DMISRS academic support at the University of Zululand					*			
3) Mokhithi (UCT): Mathematical thinking and metacognitive skills			*		*			
4) Boshoff (CPUT): Development of critical thinking				*	*			
5) Govender (NMU): Traversing the gap between Grade 12 and first year				*	*			
6) Shock (UCT): Spaghetti approach/ Curriculum reform					*			
7) Sango (UCT): Blending the equation		*				*		
8) DMISRS CoP – Assessment and mathematics education	*	*	*			*		*

3.4 Reflecting on the journey and this growing community

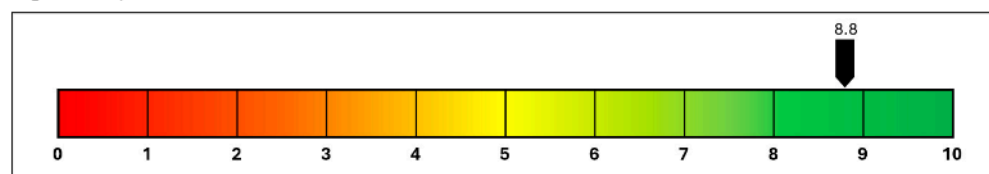
Before we concluded our discussion, we asked participants to respond to three rating questions on the Woodlap survey:

Question 8: How likely are you to continue contributing and participating in this community of practice and/or to recommend this to other colleagues?

Question 9: To what extent do you believe this community of practice and the ideas discussed here will likely impact student learning?

Question 10: To what extent do you believe this community of practice and the ideas discussed here will likely impact your professional development or professional development in this context?

Each question was presented on a 10-point Likert scale. Participants were asked to rate the likelihood or extent of impact from 0 (highly unlikely or no impact) to 10 (extremely likely or significant).

**Figure 8:** Rating results: Question 8 – Contributing and participating in CoP and/or recommending it to others

The ratings from this group ranged from 7 to 10. The average rating was 8.8 (Figure 8), which indicates that the overall feeling of the group is that they will continue to be part of this CoP, are very likely to participate in the activities and collaborative work in which the CoP will engage, and are also likely to encourage other colleagues to join and collaborate.

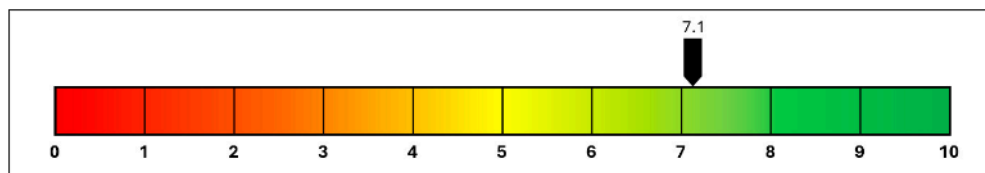


Figure 9: Rating results: Question 9 – Impact of CoP’s work on student learning

The ratings from this group ranged from 6 to 10, and the average rating was 7.1 (Figure 9), which indicates that, in general, this group thinks that the work done within this CoP – from introducing innovations and creating resources to shaping pedagogy – has the potential to have a relatively large impact on student learning.

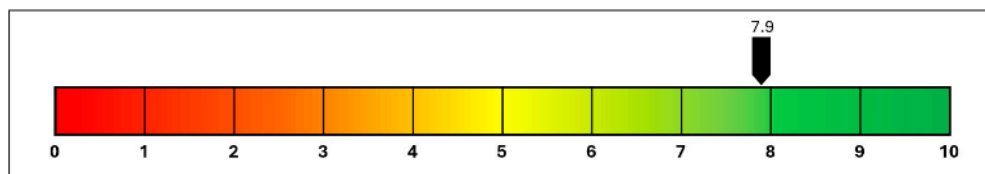


Figure 10: Rating results: Question 10 – Impact of CoP’s work on professional development

Again, the ratings ranged from 7 to 10, and the average rating was 7.9 (Figure 10), which suggests that, although comparable, the potential impact of the work of the CoP might be slightly greater on professional development than on student learning.

4. Concluding remarks

Highlighting the potential value of their research to the wider CoP and to first-year mathematics educators in general, it was evident from the panel discussion that participants could clearly see some benefit in sharing their ideas, experiences, tools and solutions, as well as their expertise in spaces such as the DMISRS project’s series of symposia. Moreover, they expressed the hope that they could retain that in some way by continuing to participate in the CoP.

Table 4: Snapshot of the responses to Question 11

Question 11: You have each contributed to the DMISRS differently. How would you describe the contribution that the innovation, solution, or example that your case study highlighted to addressing the learning needs of students, both within the first year, which was our focus for DMISRS, and beyond?

NM: My example has shown student growth regarding independent thinking and learning, and has also instilled team spirit within my students.

JS: For me, I see the potential of these types of innovations to empower students to see maths for more than just a subject that they have to pass. Through the positive changes that we hope these approaches will have on student learning, we hope that this work will also help provide students with a sense of belonging within the department of maths and mathematics as a discipline.

LB: The project I described in my case study aims to promote the development of metacognitive skills, and the ability to think critically helps students to learn better. This is important at the first-year level but is also essential at other levels of education and to mathematics education in general.

Question 11: You have each contributed to the DMISRS differently. How would you describe the contribution that the innovation, solution, or example that your case study highlighted to addressing the learning needs of students, both within the first year, which was our focus for DMISRS, and beyond?

TS: The innovation we introduced strengthens foundational knowledge in first-semester mathematics through an individualised approach, encouraging students to take greater ownership of their learning journey. Since the curriculum is also adaptive, designed to align with the demands of their specific course, it ensures that learning focuses on the most essential skills for their first-year studies. This technology provides valuable insights into students' academic profiles and behaviours, helping us better understand their learning needs.

An important idea that can be inferred from the panel's responses to Questions 11 and 12 (see Tables 4 and 5) was that the work in the CoP thus far benefits our students by directly influencing their experience of the learning process; in addition, by reflecting on how student learning takes place and what facilitates development in mathematical skills and concepts, students benefit indirectly from how this understanding informs our pedagogy and the tools that we use to shape the student learning experience, a key reason for the introduction of the DMISRS symposia as part of the overarching DMISRS project, and also highlights the rationale for continuing to work as a CoP with a specific interest in teaching mathematics at first-year level.

Table 5: Snapshot of the responses to Question 12

Question 12: The DMISRS symposia were created to share experience, solutions, and innovations in the first year of teaching and learning mathematics. How do you see this community of practice being scaled up beyond the project?

NM: Quite simply, I am of the view that DMISRS should continue for a few more years, with collaborations resulting in published work for the benefit of the wider mathematics community. Given the challenges experienced by most, a five-year project is insufficient to address the needs/issues presented during these interactions. Personally, I came on board a few years after the project commenced. I wish I had experienced DMISRS from the beginning.

FG: Perhaps maintaining the platform that the DMISRS symposia have created through attending and/or organising conference as group and sharing our work from this project with others will be a good way to continue this work.

JS: The work could also be scaled up by creating something like an online platform where we can easily share our ideas, interventions and implementations in a more "real-time" manner.

TS: We could create open-access resources and continue to promote inter-institutional research collaborations in order to build on the current momentum of the work that is being done within this community of practice. Beyond that, we could also think of developing and implementing professional development opportunities for first-year mathematics lecturers. And as for the future work of the CoP, having a focus on first-year experiences in learning and teaching, which includes student voices, as well as potentially scaling up to include international perspectives, would be important future directions to pursue.

LB: The work done through this CoP should continue to promote collaboration (across institutions) when it comes to issues such as deciding how we should teach concepts that students struggle with, as well as reviewing curricula to decide what should be included and what we could be removed. These are important questions to tackle as a CoP and not just as individual institutions or departments.

One of the biggest advantages of establishing and participating in a CoP is that, by definition, these spaces facilitate mutual engagement in attempting to address common challenges and share knowledge (Pyrko *et al.*, 2017). However, it is up to those belonging to these groups

to use the opportunities that these shared spaces create and to leverage these advantages optimally. The spaces that the DMISRS project created and the CoP that has been established will now have to find ways to continue working collaboratively and to keep the conversation going.

5. Conclusion

Based on the diverse case studies presented in the DMISRS project symposia, it becomes evident that the activities forming part of this inter-institutional collaborative project have significantly contributed to the shared practice of the Community of Practice in multiple interconnected ways, clearly demonstrating Wenger's three essential elements in action. *The domain of first-year mathematics education* has been enriched through collaborative exploration of innovative pedagogical approaches, ranging from critical thinking development through mathematical modelling (Appendix A) to integrating culturally relevant ethnomathematics perspectives using treescapes (Appendix C). *The community of educators* is strengthened through the exchange of practical solutions and resources, as evidenced by the adoption of digital tools like Gradescope for assessment (Appendix B) and ALEKS for personalised learning support (Appendix J), alongside the development of structured support systems informed by diagnostic data (Appendix L). Most significantly, *the shared practice* has evolved through collective problem-solving and knowledge creation, with institutions like UNIZULU fostering cultures of innovation (Appendix F), educators experimenting with diverse engagement strategies to address large class challenges (Appendix G), and the establishment of collaborative frameworks for continuous professional development (Appendix I). The documented improvements in student outcomes and enhanced mathematical thinking skills to better transition support demonstrate how Wenger's typical CoP activities of collaboratively tackling key problems, seeking knowledge, sharing experiences, and coordinating efforts have been successfully realised, while also highlighting the urgent need for a sustained platform that enables CoP members to systematically share these emerging practices, learn from each other's innovations, and coordinate combined efforts that can achieve wider-reaching impact across the South African higher education system and beyond.

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Appendix A: Designing an activity to support the development of critical thinking

Case study author: **Liezie Boshoff (CPUT)**

Dr Liezie Boshoff (LB) has lectured on applied mathematics in the engineering faculty for several years, focusing on students' critical thinking and problem-solving. To support this, she designs activities to enhance these skills. This case study explores how a mathematical modelling activity fosters critical thinking among first-year engineering students.

Abstract

Engineering Mathematics 1 (EMA154X) is a first-year course with 20 students, featuring two 90-minute weekly lectures and one 90-minute practical session. The course integrates mathematical modelling, focusing on developing critical thinking (CT) skills. With a strong interest in teaching CT and mathematical modelling, my approach explores how problem-solving activities and practical applications can enhance students' analytical abilities and mathematical reasoning.

1. Introduction

Engineering lecturers often complain that their students have a silo mentality and that they struggle to think critically. Although lecturers are aware of the problem, they find it challenging to provide their students with the opportunity to develop their critical thinking (CT), creativity and graduate attributes. I teach first-year mathematics in an engineering faculty, where mathematics is a service subject. My primary goal is thus to ensure that students learn the mathematics needed to engage with their other subjects and future careers. Mathematics syllabi are so overloaded that there is not enough time to properly illustrate the use of mathematics in class, let alone provide students with the opportunity to develop their CT skills.

Nevertheless, an important purpose of higher education is to develop students' CT ability. Not only does this ability aid learning, but CT also serves as preparation for an uncertain future. Curriculum reform focused on improving the value of subjects in engineering qualifications is gaining momentum, so when I found myself teaching a relatively small class (20 students) who were registered on the extended curriculum programme – which allows for more contact time with students – I reasoned that this was the ideal learning environment for the development of students' CT skills and implemented an activity to provide students with the opportunity to develop these skills. This is how I did it.

2. Solution

Firstly, I surveyed the literature to find out exactly what CT was, and what activities were most suitable for developing engineering students' CT skills. Most importantly, I found that:

- Brent and Felder (2014) advise that students learn to think critically in the same way as they learn any other skill – through practice and feedback; according to Niss and Blum (2020: 91), students learn modelling in large part by actually doing modelling.
- I did not need to design a framework to construct an activity that enables engineering students to become critical thinkers – such a framework had already been designed by Ahern et al. (2012), who point out that CT is a “movement from the concrete, or factual,

to the abstract – and back again”. It is the ability not only to use knowledge and facts to create ideas, concepts and solve problems, but also to use these developed concepts, theories and ideas in the real world.

- According to Geiger et al. (2022), mathematical modelling entails “identifying a problem within a real-world context, developing a suitable mathematical representation, determining a subsequent mathematical solution, interpreting the solution within the original context, and evaluating the solution’s validity for resolving the problem”. Thus, as described by Geiger et al. (2022), mathematical modelling is a special case of CT, as described by Ahern et al. (2012).
- Abassian et al. (2019) identify various perspectives on mathematical modelling. For this study, a realistic perspective is taken, which means that the focus is on pragmatic–utilitarian goals – students work with realistic, authentic and messy tasks to develop the ability to solve real-world problems.
- In the context of engineering education, model-eliciting activities (MEAs) are open-ended, real-world, client-driven problems which focus on the process rather than the product, thereby providing the opportunity for the development of students’ higher-order thinking skills (Diefes-Dux et al., 2004).

The literature convinced me that an MEA would be suitable for the development of CT, but I still needed an MEA suitable for my students’ abilities. Niss and Blum (2020) provide many sources for finding suitable material. However, I found that De Villiers (2018) had recently investigated the concurrent development of mathematical modelling and the engineering technician competencies of first-year engineering technician students. In her study, students engaged with various MEAs suitably chosen for their abilities. The participants in her study had no prior experience with mathematical modelling and were registered for an extended education programme at a university of technology, like my students.

For authenticity, I modified one of the modelling activities that De Villiers (2018) employed in her study. The modified problem required that students find the position of a visitor to their campus regarding the outcome of their COVID-19 screening test. The information provided was that the visitor was not answering their cell phone and that three cell phone towers (a minor freestanding cell mast and two minor rooftop masts attached to the rooftop corners of well-known buildings on campus) had just detected signals from the phone from distances which were specified. It was expected that students would investigate and then estimate the heights of the mentioned types of cell phone towers and well-known buildings on their campus and then employ both Pythagoras’s theorem and their knowledge of circles (knowledge which they had previously acquired but would not necessarily relate to the problem) to solve this problem.

I also invited the small group of lecturers and technicians who taught the same students as me to get involved. Together, we decided that we would present the students with an integrated project, where lecturers would each control how they evaluated their students’ performance with the graduate attribute assigned to their subject, that students would take part in teams of four people each, and that they would present their projects face-to-face with PowerPoint. Subject lecturers assessed their own students’ work according to the graduate attributes assigned to their subject – communication (professional and technical communication and independent learning ability), physics (investigations, experiments and data analysis) and mathematics (problem-solving).

3. Reflections

In summary, our students had a positive experience with mathematical modelling, which helped them understand how mathematics plays a role in their day-to-day lives. About CT, various stages of the modelling cycle offered students support for learning to think critically – they reflected on their thought processes and those of their peers when they exchanged feedback, and when their lecturers critiqued their thought processes during their PowerPoint presentations. Participating lecturers reported that the activity broke down siloes in which they and their students often worked; that they were particularly impressed by how much students learnt through the process; that they appreciated the creative ways in which groups solved the problem; and that the students produced high-quality PowerPoint presentations.

4. Observed change

The learning that had taken place was different and not measurable in terms of the usual “how much”, but rather in terms of “what” was learnt. Qualitative feedback from the lecturers thus served in place of a quantitative test that measured the improvement of students’ CT skills. Although it was time-consuming to develop and employ a mathematical modelling activity, and more challenging to provide feedback on a CT activity (compared to the work that our students normally submit), the following development would not have been possible in conventional lecturing environments. We were pleased with how much was gained in the process. We hope to make mathematical modelling a permanent part of the extended curriculum programme. The development of students’ CT through mathematical modelling activities does nevertheless require that a proper infrastructure, time and resources be put in place, and also that the curriculum be developed to accommodate change.

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Appendix B: How Gradescope revolutionised my marking process

Case study author: Anita Campbell

Prof Anita Campbell (AC) is an educator and researcher dedicated to enhancing the success of mathematics students. Since 2011, she has taught engineering mathematics in UCT's academic support programme for engineering. Her 2020 PhD in engineering education focused on growth mindsets. Her research explores positive psychology in university mathematics, addressing failure, character strengths, trust, group work, and growth mindset in assessment. She enjoys sharing her experiences using Gradescope because happiness is meant to be shared.

Abstract

Gradescope has transformed my grading process, making it more consistent, flexible, transparent and faster. The platform supports multiple markers, enables quick correction of errors, and allows tailored release of grades and feedback. Its features also enhance marker up-skilling and quality through peer review and accountability, enabling students to request re-grades and track personalised feedback. Gradescope has made my grading more efficient and helps to promote learning from mistakes.

1. Introduction

Grading can be daunting for educators, particularly in ensuring consistency and transparency. Gradescope³ is an online grading platform that uses AI-assisted and manual grading to streamline assessment for instructors and students.

2. Solution

Since I started using Gradescope, my marking process has become more consistent, flexible, transparent and faster. This is how I use it.

- I implemented Gradescope to allow multiple markers to mark simultaneously, even on the same question.
- Gradescope empowers less experienced markers to upskill through meetings and peer review of the marking by all markers.
- I use the platform's features to correct marking errors swiftly.
- I leverage Gradescope's accountability features to track the marking team's progress and enhance quality.
- I save grading time by selectively marking questions in tutorial submissions or using submissions as a register.
- I enabled "regrade requests" so that students can address inconsistent or incorrect marking.
- I track student engagement with feedback to evaluate assessment effectiveness.

³ <https://www.gradescope.com/>

3. Reflections

The platform has significantly streamlined the grading process, making it more manageable and efficient. The flexibility and transparency provided by Gradescope have greatly improved the grading quality.

4. Observed change

Gradescope has revolutionised my marking process, leading to faster, more consistent grading and improved quality of assessment. Since students know that I can easily tell if they have looked at their graded scripts, they have more incentive to do so. Since it is easy to see where marks were lost, misconceptions can be uncovered and resolved. I love that unprofessional-looking piles of pen-graded uncollected scripts no longer litter my office or public spaces.

Appendix C: Exploring the treescapes concept for teaching mathematics: Ethnomathematics perspective

Case study authors: Ekaterina Rzyankina and Frikkie George

Dr Ekaterina Rzyankina (ER) is part of the Global Institute for Teacher Education and Society and a lecturer at the Cape Peninsula University of Technology.

Dr Frikkie George (FG) lectures at the Cape Peninsula University of Technology (CPUT) in Cape Town, South Africa. He is responsible for mathematics support at the Fundani Student Learning Unit, a support unit at CPUT.

The authors have been working together on an innovative new educational approach that seeks to integrate the concept of “treescapes” into teaching mathematics. Trees play an important role in African cultures and can be a useful subject and a metaphor for instruction.

Abstract

The indispensability of trees in our ecosystem and their profound resonance within African cultures are irrefutable. As urbanisation and modern lifestyles escalate, so does the demand for tree resources, leading to an alarming increase in market value and a concerning surge in deforestation. This case study introduces an innovative educational approach, which integrates the concept of “treescapes” – the visual and spatial arrangement of trees within a landscape, encompassing their composition, structure and aesthetic qualities – into mathematics education.

1. Introduction

In our mathematics classroom, “treescapes” visually represent hierarchical data like family trees. The concept of treescapes emerges as a powerful visual tool. These treescapes elegantly depict hierarchical structures, allowing us to navigate complex relationships readily. Whether representing family trees or mathematical principles, treescapes provide a tangible framework for understanding inheritance, evolution and classification. The study also recommends using trees to help students visualise relationships between variables and functions and understand the logic behind abstract mathematics concepts.

2. Solution

Practical applications extend to outdoor activities where students engage with natural treescapes, examining patterns like the Fibonacci sequence and developing their investigative skills. The study also explores how tree branching patterns can parallel concepts in electrical circuits, chemical bonding and various branches of mathematics and computer science.

Exploring the concepts of treescapes delves deeper into trees’ historical and cultural significance in African societies, exploring how this rich heritage can be leveraged in educational contexts. Students also examine traditional African knowledge systems and their connections to the natural world, highlighting how these perspectives can provide a more nuanced and comprehensive understanding of mathematical concepts. This exploration includes case studies and examples from various African cultures, illustrating the diverse ways in which trees have been integrated into traditional knowledge systems and practices.

3. Reflection

This mathematics approach of using the concept of treescapes not only examines the integration of the treescape concept in teaching practices but also employs a critical methodology to interpret the African perspective on trees, highlighting their multifaceted roles in education. Trees' positive impact on student learning positions them as dynamic educational tools that bridge the natural world and mathematical principles.

4. Observed change

The concept of treescapes is further expanded to include its application in environmental education and sustainability. In our case study, we noticed how teaching about trees and their ecosystems can foster environmental awareness and stewardship among students. During the practical engagement, students discussed the role of trees in addressing global environmental challenges, such as climate change and biodiversity loss, and how this can be integrated into mathematics and STEM education to promote a more sustainable future. This interdisciplinary method enriches the learning experience and nurtures a deeper connection between students and their environment, encouraging a more holistic understanding of the world.

Appendix D: Traversing the gap between Grade 12 mathematics and first-year mathematics: My first-year personal reflections

Case study author: Vasuthavan Govender

Dr Vasuthavan Govender (VG) is a mathematics education specialist who has worked in the field for over 40 years as a mathematics teacher, college lecturer, subject advisor, university lecturer and chief education specialist: teacher development. His research interests are mathematics teaching and learning, problem-solving and mathematics Olympiads, and the gap between school mathematics and university mathematics. He is a research associate at Nelson Mandela University and lectures on third- and fourth-year mathematics method modules.

Abstract

This case study explores challenges faced by first-year students transitioning from Grade 12 to university mathematics, drawing on the author's experience teaching mathematics service modules in the Eastern Cape. Key factors include lesson planning, student backgrounds, assessment practices and instructional strategies. The findings emphasise the need for targeted teaching approaches and better alignment between school and university mathematics.

1. Introduction

This case study took me back to when I was lecturing on mathematics service modules at a university in the Eastern Cape. I shared my experiences in teaching these modules, covering lesson planning, knowing your students and teaching students from diverse backgrounds, assessment practices and other key details that may have contributed to an enriching experience for students in these modules, thus paving the way for university success. From my own experiences, as well as an examination of mathematics courses offered at various universities, I have classified the mathematics offered at universities into three broad groups:

1. Mathematics and/or applied mathematics courses are taken as a major in a faculty of science undergraduate degree programme. These may be taken up to the third-year level and provide access to post-graduate studies involving specialising in a field or branch of mathematics.
2. Specialist mathematics courses are mostly linked to programmes in the faculties of science or engineering but may also form part of advanced programmes in commerce and education. These courses may include engineering mathematics, actuarial mathematics, mathematical statistics, mathematics (for teaching), and others. These courses may go up to second- or third-year level as part of first-degree programmes or be prerequisites for other mathematics-related graduate and post-graduate studies.
3. Service mathematics courses may be taken in most faculties, such as science, commerce, health sciences, education and engineering. These courses are usually offered at the first-year level and may comprise one or two modules. Engineering mathematics is likely a specialist mathematics subject, while the others could be classified as service mathematics subjects.

1.1 Grade 12 mathematics

NSC mathematics papers have been fairly accessible, all set according to strict curriculum requirements. Table 1 shows the current allocation of levels of thinking for each of the Grade 12 mathematics papers.

Table 1: Grade 12 mathematics: Cognitive levels

Levels of thinking	Suggested %
1. Knowledge	20%
2. Routine procedures	35%
3. Complex procedures	30%
4. Solving problems	15%

Most university courses for which mathematics is a prerequisite require a minimum of 60% (with some going as high as 80%). This means that, even if a learner gets full marks for all level 1 and level 2 questions, the learner must be able to get some marks in level 3 and possibly level 4 questions to achieve a minimum of 60%. However, year after year, the examiners' reports for mathematics (both papers) show that while learners can do well in level 1 and level 2 questions, they tend to falter with questions involving a higher cognitive demand. Therefore, learners at the school level need more exposure to level 3 and level 4 questions.

2. Solution

2.1 2022 survey of mathematics teachers

In August 2022, I surveyed mathematics teachers in three districts (one urban and two rural) of the Eastern Cape. These teachers were involved in a series of problem-solving workshops. The survey was conducted in preparation for the DMISRS symposium in September 2022. Teachers were asked what advice they would give to first-year university mathematics lecturers to bridge the gap between school and university. Twenty teachers participated in the survey and responded as follows:

Be enthusiastic and always give support to these new learners. Lecturers must know the Grade 12 syllabus to understand what learners should know.

They must give them more mathematics work, assign tutors, and ensure that students write weekly tests.

Lecturers should start from the known, ensure the foundation is unshakeable, and then move straight to the unknown. They will "ace" it.

Lecturers or university representatives should visit schools and have talks with Grade 12 learners on preparing for university programmes and possible careers involving mathematics.

There should be extended mathematics programmes to bridge the gap between school and university.

Lecturers should not take things for granted; sometimes we teach students just to get good marks, and they tend to forget their work.

Lecturers should instil a love of mathematics in students.

They should adopt a developmental approach by linking the work with Grade 12 mathematics.

There should be some revision of key topics from school before going into the “university” content of their modules.

The survey of mathematics above prompted me to reflect on my own experiences when teaching first-year students in the subjects mentioned earlier. These reflections form the basis of this case study and are listed as follows:

- When planning lessons or a series of lessons, always be sure what you want your students to know, understand and use in your subject and other subjects in their programmes, such as the ability to do basic calculations, to do work that requires higher-order thinking, to be involved in analytical and critical thinking, etc.
- While university lecturers may try to expose students to high-level mathematics to show them that university mathematics is “difficult” and different from school mathematics, adopting a developmental approach in the first year tends to be more inclusive and gives more students a chance of success.
- It may be prudent for first-year university lecturers to get to know their students. Students come from various backgrounds: schools in affluent areas, schools in poor areas, schools in rural areas, township schools, etc. The one common factor for all the students in our classes is that they have fulfilled the requirements for studying the university programme (degree or diploma), and “they deserve to be in our class”.
- Assessment practices and teaching go hand-in-hand. The assessment of the module must be closely aligned with the teaching.

3. Reflections

The transition from school to university in South Africa has major implications for the higher education sector. In programmes for which school mathematics is required and where mathematics is likely to feature, a lot depends on teaching and learning in these first-year classes. My experiences and reflections may offer a glimpse of what is possible in these classes.

4. Observed change

Key improvements in first-year mathematics instruction included: 1) greater awareness among lecturers of the nature of the school–university gap; 2) increased use of developmental approaches to aid student success; 3) more interactive teaching and continuous assessment to enhance engagement; and 4) recognition of structured transition programmes as essential for first-year mathematics students.

These findings underscore the importance of informed and adaptable teaching practices in supporting first-year mathematics students.

Appendix E: Reflections on support and assessment of engineering mathematics during COVID-19

Case study author: Darlington Hove

Dr Darlington Hove (DH) is a mathematics and basic numeracy lecturer at the Mangosuthu University of Technology. Most of his students study engineering, and he is therefore interested in teaching and assessing mathematics for engineering.

Abstract

This case study examines the impact of emergency online teaching on first-year engineering mathematics students at MUT, highlighting the disparities between advantaged and marginalised students. With 68% of the student body being first-generation university students and 90% reliant on NSFAS funding, challenges such as limited connectivity and resources shaped their learning experience. The study reflects on teaching approaches, assessment strategies and student feedback, contributing to ongoing research critiquing the suitability of existing online practices for undergraduate engineering mathematics.

1. Introduction

The findings presented in this case study are linked to ongoing exploratory research work, which aims to critique the suitability of existing online teaching practices for undergraduate engineering mathematics by reflecting on the experience of teaching, as well as analysing the student voice. This research involves a first-year undergraduate engineering mathematics module at MUT, spanning three departments with 220 students. Sixty-eight percent of first-generation university students are largely from disadvantaged backgrounds and poorly resourced high schools. Ninety percent rely on NSFAS funding, and most are challenged regarding access to connectivity, gadgets and data.

With the switch to online teaching during the pandemic, we observed that the difference in the experiences of our students from marginalised communities, compared to those from more advantaged backgrounds, led to great discrepancies in their learning experience. At the time of the research, students were going through a class rotation system, which initially consisted of one month on campus and one month spent on remote learning. It was then adjusted to one week on and one week off. Remote learning was conducted using MS Teams, and communication channels such as WhatsApp, email and the Blackboard learning management system. Assessments shifted from a summative focus to continuous assessments conducted during on-campus periods. Students' access to online learning varied:

- Students staying on campus had access to computer labs and the university network.
- Students staying in off-campus residences had access to data.
- Students staying in private residences had access to the network and data.
- Students staying in rural areas had no access to the network.

For the research, a tool was designed and circulated to students to gather their reflections on their experiences with the class rotation system. This research drew on the APOS framework by Dubinsky (1991): mathematics students solve mathematical problems by progressively constructing mental action, processes and objectives, and synthesising them into schemas to make sense of situations.

Some of the issues experienced in online teaching and learning in this context included the following:

- Online attendance was a problem when there was a delay in the university's provision of data.
- At least 50% of students attended during class, but a small number participated.
- Online classes are often disrupted when students unmute themselves or try to record a lecture.
- Attendance dropped during periods when project reports were due. While this was already an issue before the pandemic, it worsened with remote learning because students felt they could simply record the lecture and watch it later.

From the reflection tool, the following feedback from students was collected:

- Students acknowledged the benefits of online teaching but indicated that concentration was a challenge due to competing priorities in the absence of the physical presence of the lecturer.
- The primary sources of distractions were social media (especially WhatsApp) and Facebook videos.
- Many students preferred contact lectures over online lectures, although many preferred a mix.
- Students found it "off-putting" when lecturers taught using online slides, preferring that lecturers write things on the board. When slides were given to students before the lecture to flip the classroom, students felt they were beneficial.
- Students preferred solving problems together in class to benefit from group participation.
- Many students were experiencing online learning for the first time and found it confusing, especially when lecturers only sent out slides.

2. Reflections

Given the objectives of this specific course as a service module, this mode of delivery introduced challenges that were not ideal for meeting the goals. Learning mathematics online presents challenges in cases where students must rely 100% on the online model, and limited access to devices and connectivity, as well as limited knowledge of technology, worsen the experience. Slides were not conducive to learning as there was too much content to grasp, or the students became too passive, planning to read the slides later. Rather, using devices that allowed for writing (e.g. tablets) seemed to connect students with the lecture and stimulate greater interest and conversation.

3. Observed change

Implementing emergency teaching measures during the COVID-19 pandemic disproportionately affected advantaged versus marginalised student communities. The findings highlighted significant challenges in emergency online teaching, including accessibility barriers, low engagement, and traditional online content delivery limitations. Choosing these devices should be considered from the students' point of view, not only the academics'.

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Appendix F: Summary of the activities that the University of Zululand has done with the DMISRS project

Case study author: Syamala Krishnannair

Prof Syamala Krishnannair (SK) is an applied mathematician, and her field of research is in process data analytics.

Abstract

This case study outlines UNIZULU's engagement with the DMISRS project and collaboration with the Centre for Educational Assessments (CEA) team to enhance first-year mathematics teaching and learning. Workshops and symposia addressed challenges such as student preparedness and large class sizes. Initiatives included NBT administration, diagnostic assessments and adaptive learning strategies. Despite disruptions caused by COVID-19, ongoing collaboration has informed innovative teaching practices and assessment strategies, aiming to improve student success in calculus.

1. Introduction

I attended the DMISRS symposia in 2018 and 2019. In 2019, I participated in the panel discussion on "Mathematics for science", highlighting the challenges faced by UNIZULU students, such as school background (99% are from a rural background), class sizes, etc. Student support systems are available, including in-person tutorials and online material distribution using Moodle. However, we must reflect on teaching practices and assessment strategies to promote student success.

2. Solution

My engagement with the DMISRS project has been a journey of learning and adaptation, shaped by the challenges and realities of teaching mathematics at UNIZULU. When I first attended the DMISRS symposiums in 2018 and 2019, I gained valuable insights into how other institutions used NBT data to inform their teaching strategies. These symposia inspired me to explore how we could apply similar approaches at UNIZULU, given our unique student demographics and their difficulties in transitioning to university mathematics.

In 2019, the CEA team visited UNIZULU to facilitate a workshop that allowed us to reflect on our teaching challenges and explore the potential of using diagnostic assessments. We examined our students' NBT profiles and discussed using this data to enhance our support structures, leading to a significant milestone in 2020 when we administered the NBTs to nearly 500 students, giving us a clearer picture of their strengths and gaps. The plan was to follow up with tailored support materials and academic workshops, but COVID-19 disrupted everything. The shift to online learning posed immense challenges, from network connectivity issues to difficulty keeping students engaged remotely.

Despite the setbacks, we kept moving forward. In mid-2021, we conducted an online workshop with the CEA team, where we revisited our goals and explored how diagnostic data could be used to understand first-year mathematics performance better. By 2023, our discussions had evolved to include adaptive learning initiatives. Seeing how these tools could personalise learning experiences gave me renewed hope for improving student outcomes.

3. Reflections

My participation in the annual DMISRS symposium and collaboration on the project helped me to apply innovative teaching practices and assessment strategies discussed by the different themes in the symposium. The intention is to use these efforts to improve the pass rate in the calculus course at UNIZULU.

4. Observed change

UNIZULU's engagement with the DMISRS project has fostered a culture of innovation, encouraging new approaches to first-year mathematics education and student support. With a strong commitment to making a difference, UNIZULU will seek funding for a further collaborative research project to explore strategies for enhancing student success in mathematics.

Appendix G: It's hard work keeping the attention of a distracted audience

Case study author: Narenee Mewalal

Ms Narenee Mewalal (NM) is an applied mathematician at a university of technology. While her research background is in astrophysics and cosmology, her extensive experience in high school mathematics teaching and management has led her to focus on undergraduate mathematics education. She is particularly invested in supporting first-year students, many first-generation university entrants from low-income backgrounds and under-resourced schools. Recognising the challenges posed by digital deficiencies and the transition to university, she prioritises academic and technological mentoring. By fostering a more supportive and engaging learning environment, she aims to build student confidence – an essential foundation for success in mathematics.

Abstract

There are 150 students in my class. I see them four times weekly for 50-minute lectures and one tutorial session. For this group of students, mathematics is a one-semester terminal module. The case study highlights the various challenges experienced. It examines some of the teaching strategies I have implemented to foster interest in the subject and improve success rates. The outcome significantly improved students' positive self-concept and pass rates.

1. Introduction

The cohort of first-year students referenced in this article is registered for a degree in life sciences. They are of diverse backgrounds and have varying levels of ability. However, a common feature is that many first-generation university entrants come from low-income families and poorly equipped schools. My experience has been that, to overcome the many distractions of first-year tertiary education, these students tend to “sacrifice” mathematics, viewing it as less important because it is a non-major module. So, to ensure that my teaching reaches all, I decided first to change mindsets and then work on mathematics. A description of my strategy follows.

2. Solution

I employ a blended approach, teaching both face-to-face and using available online learning platforms in the following way:

- Because of the digital divide, I start my first lecture by teaching students the basics of login details, the learning management system (LMS) structure, Moodle, navigating it, and what a Microsoft Teams classroom looks like. Immediately, I capture their attention and interest.
- As the class size does not allow me to gauge individual abilities, I focus on school revision in the first two weeks to bring them all to a common starting point.
- While I recommend a list of textbooks for additional reading, I prefer to develop the learning material for this module, which I upload to the Moodle classroom. This allows students to read ahead of a contact lecture or to reinforce concepts discussed in class at a pace with which they are comfortable.
- During student protests, I resort to recorded video lectures and live online lectures via Teams, which I also record for the benefit of all.

- Likewise, I maintain a multimodal approach with minor assessments completed online and major assessments written in-person on campus. For the online assessments, I use the randomised function of the LMS to generate quizzes, thus ensuring the integrity of the testing instrument.

3. Reflections

In summary, by providing my students with various consolidation mechanisms, I created a classroom environment that was inclusive, more personalised and more conducive to learning.

4. Observed changes

A significant growth in students' positive academic concept was observed, resulting in increased classroom interactions and, importantly, a noteworthy improvement in pass rates from 45% in 2019 to 87% in 2023.

Appendix H: Enhancing mathematical thinking and metacognitive skills in a first-year mathematics course

Case study author: Mashudu Mokhithi

Mr Mashudu Mokhithi (MM) is a lecturer in the Department of Mathematics and Applied Mathematics, Faculty of Science, University of Cape Town. He has a particular interest in tertiary mathematics education. He is part of a team of colleagues working on mathematical thinking workshops for first-year mathematics students, and his case study focused on this work.

Abstract

MAM1033F is a first-year mathematics course with 50 to 60 students, combining traditional lectures with interactive workshops and tutorials. Students attend daily 45-minute lectures, two two-hour workshops per week, and a weekly two-hour tutorial. The course enhances mathematical understanding through structured engagement, integrating language support and worksheet-based activities. A team of lecturers, facilitators, and experts collaboratively develop and delivers the curriculum, ensuring a comprehensive learning experience that supports diverse student needs.

1. 1. Introduction

The Courses Impeding Graduation (CIG) annual report of 2019 identified MAM1000W (first-year mathematics course for science students) as one of the courses with a worryingly high failure rate. MAM1000W is a prerequisite for many second-year courses; if students fail this course, their degree will likely be extended by at least one year. There have also been concerns about the pass quality among the students who manage to pass MAM1000W. After a lengthy and detailed curriculum redesign, starting in 2023, MAM1000W was divided into two one-semester courses. Each of these comes in a regular and an augmented flavour, where mathematical thinking workshops are given. The four new courses are MAM1031F/MAM1033F (first semester, regular and augmented, respectively) and MAM1032S/MAM1034S (second semester, regular and augmented).

The high failure rate in first-year mathematics can be attributed to many factors, including the challenges students face when transitioning from high-school mathematics to university mathematics. Most first-year students arrive at UCT without the mathematical thinking and metacognitive skills necessary for success in first-year mathematics.

2. Solution

In order to address these challenges and foster a successful transition from high school to university mathematics, we implemented various interventions in the form of regular workshops. Although the workshops cover mathematical content, they mainly aim to enhance the mathematical thinking and metacognitive skills of first-year students. The class consists of 50 to 60 students. The course offers two two-hour workshops on Monday and Friday afternoons, daily 45-minute traditional mathematics lectures, and a weekly two-hour tutorial. The workshops incorporate various instructional approaches, such as problem-solving activities, collaborative learning and reflection exercises. These approaches encourage students to engage in critical thinking, to analyse mathematical problems from multiple perspectives, and to develop effective problem-solving strategies.

Additionally, the workshops emphasise metacognitive strategies, including goal-setting, self-monitoring and self-reflection, to enhance students' awareness of their thinking processes and to promote strategic learning. To ensure the effectiveness of these workshops, continuous evaluation and improvement processes have been implemented. Feedback from student focus groups and written reflections is collected and analysed to identify enhancement areas and inform the design and implementation of future workshops.

3. Reflections

Gathering feedback about the workshops has provided valuable insights into students' experiences and the impact of the intervention. Several key observations and insights have emerged through ongoing reflection and feedback from students.

Firstly, students have expressed a greater awareness of their thinking processes and a deeper understanding of the importance of metacognition in mathematical problem-solving. They have reported that the workshops have equipped them with the tools and strategies to approach mathematical exercises strategically. Some students reported a heightened self-introspection, which has empowered them to take more active roles in their learning and has resulted in improved confidence in tackling challenging mathematical exercises.

Additionally, students have highlighted the value of collaborative learning and peer interaction in the workshops. They added that engaging in group problem-solving activities has allowed them to benefit from diverse perspectives and shared strategies, and to learn from one another's approaches. The collaborative environment has fostered community and support, enabling students to overcome challenges collectively and build stronger problem-solving skills. Moreover, students said they enjoyed including hands-on activities, making the learning experience more relevant and engaging. Students have reported a deeper understanding of mathematical content.

However, it is important to identify challenges and areas for improvement. Some students have expressed the need for guidance or solutions to the workshop exercises, although they acknowledge that the point of the workshops is not to get the right answers but to understand how to get there. Addressing this concern may involve providing clear written solutions to the workshop exercises. There have also been concerns about ventilation in the venue and distractions by students who are not part of the workshops. These issues will be addressed in the next semester.

4. Observed changes

- Mathematical thinking and metacognitive skills workshops have effectively improved students' problem-solving abilities and promoted self-awareness in the learning process (this is from students' feedback).
- Collaborative learning and peer interaction play a significant role in enhancing students' mathematical thinking skills. The workshops provide a supportive community where students can learn from one another's perspectives and approaches (also from students' feedback).
- Ongoing reflection and student feedback are essential for refining and improving the workshop design. Addressing students' concerns can further enhance the effectiveness of the workshops.

- Overall, the case study highlights the significance of incorporating mathematical thinking and metacognitive skills training in educational settings.

Here is an extract from a focus group interview:

I think the workshops for me were very exciting. Every session that I attended. I think it was worth it because it gives us a different flavour of learning. The way the facilitators attend to us and the way they help us realise how useful it is to be conscious about how you're learning mathematics. So that was very impactful because learning mathematics became something that's beyond just sitting down on your study desk and trying to solve problems and getting stuck and all that, but you become very conscious of the day by day. The journey of actually priming up your skills, of actually working through certain things that you didn't understand. And the problem-solving there is really good because they give you a large variety of problems, and they teach us how to think. So we are learning how to learn, which is something you don't really get anywhere in the university. We don't really get that in tutorials. We don't really get that in lectures also because there's little time, and we are just getting the content.

Appendix I: A community of practice teaching and learning mathematics: Collaboration, collegiality and collective reflection

Case study author: Pragashni Padayachee

Prof Pragashni Padayachee (PP) is an associate professor in mathematics education at UCT's engineering support programme, with over 30 years' experience in higher education. Her research spans mathematics education, engineering education and academic development, focusing on support initiatives that enhance student participation and success. Now in her second term as chair of the USAf teaching and learning mathematics community of practice (USAf TLM CoP), she leads strategic direction, collaboration, national projects and advocacy for mathematics education. She strives to build a vibrant, inclusive community that champions excellence in teaching and learning.

Abstract

This case study highlights key projects and focus areas of the USAf teaching and learning mathematics community of practice (TLM CoP). Comprising mathematics and mathematics education colleagues from South Africa's 26 higher education institutions, the group engages through regular meetings, symposia and research collaborations. Led by Assoc Prof Pragashni Padayachee, its priorities include enhancing the first-year mathematics experience and advancing mathematics teacher development. The CoP fosters national collaboration to improve mathematics teaching and learning in higher education.

1. Introduction

The TLM CoP was established by USAf in 2016 to address the expressed need in the university sector for a platform where mathematics lecturers and practitioners can collaborate and share good practice and resources. Its mandate is to promote and strengthen teaching, learning and research in mathematics, contributing to the transformation needs of South Africa. This CoP exemplifies collegiality, collaboration and collective reflection. Mathematics and mathematics education academics join forces to exchange knowledge, experiences and resources, fostering a sense of mutual respect and support. Together, they strive to develop innovative teaching strategies, design engaging learning activities and tackle common challenges. This environment encourages critical conversations and reflections on their own practice while benefiting from the insights of others.

2. Solution

Project 1: First-year interventions to improve the teaching and learning of mathematics (Lead: Dr Mark Jacobs, CPUT)

Students entering higher education institutions (HEIs) who intend to follow a course of study in STEM disciplines must do at least first-year mathematics. Students' difficulty navigating this academic transition from high-school mathematics to first-year mathematics is well documented. Academics also find this difficult terrain because of the students' uneven school mathematics preparation. There is increasing concern in HEIs about first-year mathematics performance and success. A negative first-year mathematics experience increases dropout rates, extends the time to complete a degree, and may impede graduation. This research will be conducted on first-year mathematics at a whole sector level. A comprehensive picture of the sector may influence mathematics teaching and learning policy and practice nationally and potentially inform the teaching and learning of mathematics at HEIs in South Africa.

Project 2: Teacher development project (Lead: Professor Vimolan Mudaly, UKZN)

The TLM CoP identified the need to review the training of mathematics teachers in universities and to improve the in-service training of current mathematics educators. The objectives of this project are to explore the quality of the mathematics teacher education programmes and the teaching practice experiences offered by universities nationally and internationally, and to revitalise and strengthen the nature of programmes at universities so that the teacher education system becomes a powerful driver in improving educational standards.

3. Reflections

As a CoP, we have an incredible opportunity to share our challenges, experiences and solutions, to interrogate our own and each other's practices, and to ultimately benefit the teaching and learning of mathematics. As a CoP, we undertake to maximise such opportunities. These are some key issues and endeavours featured in our collaboration and conversations.

3.1 Pedagogy

Within the TLM CoP, educators discuss and share pedagogical strategies that promote effective mathematics instruction. They exchange ideas on how to make the subject more accessible, engaging and relevant to diverse learners. Through collective and critical reflection, they evaluate different pedagogical approaches and adapt them to meet their students' specific needs and contexts.

3.2 Technology for learning

This CoP embraced technology in mathematics teaching and learning, and educators shared knowledge on effectively integrating digital tools, interactive platforms and virtual resources to enhance student engagement, problem-solving skills and mathematical reasoning. We have noted the enormous potential for using technology to deepen and support learning in the mathematics "classroom". The pandemic has illuminated what can be done with this online space: engaging, enriching and accessible. Videos and interactive media are now part of how students learn, and discussion boards allow conversations to continue and for ideas to be recorded outside of classes.

3.3 Assessment

Assessment was another crucial aspect addressed in this CoP. Educators collaboratively developed assessment methods that align with their instructional goals and foster meaningful learning experiences. They shared insights on formative and summative assessment techniques. During the COVID crisis, they shared open-book assessments and encouraged assessment as learning, where assessment is part of the learning journey.

3.4 Multilingualism colloquium

The TLM COP recognised the importance of multilingualism in teaching and learning mathematics; it emphasised that students' diverse linguistic and cultural backgrounds should be acknowledged and highlighted the value of incorporating their languages as resources for learning.

An upcoming colloquium is planned to explore the intersection of multilingualism and mathematics education. The colloquium aims to identify and develop innovative teaching strategies that leverage the strengths of multilingualism and inter-disciplinary learning to enhance the quality of mathematics education. By fostering inter-disciplinary dialogue and collaboration, the colloquium seeks to advance knowledge and understanding of how multilingualism can be effectively integrated into mathematics education, ultimately benefiting students and educators alike.

3.5 Crises

During crises such as the COVID-19 pandemic, the CoP became even more critical. Educators came together to address the challenges of remote or hybrid learning models, sharing strategies, resources and technological solutions to ensure continuity in mathematics education. They collaborated on adapting instructional methods, supporting students' socio-emotional needs, and leveraging online platforms to facilitate interactive and collaborative learning experiences.

4. Observed change

The TLM CoP provides a platform for educators to share knowledge, resources and experiences; and to work together to develop innovative pedagogical strategies, enabling educators to analyse their practices critically, adapt to diverse student needs and continually improve their pedagogy. By embracing these principles, the TLM CoP enhances professional growth, promotes student success, and contributes to advancing mathematics education.

Appendix J: Blending the equation: The final year of the DMISRS project

Case study author: Tatiana Sango

Ms Tatiana Sango (TS) has a background in applied mathematics and experience in teaching and educational publishing; she has developed print and digital resources for high school and first-year mathematics students. Now working with high-stakes assessments, she recognises the varied mathematical foundations that students bring to university and how adaptive learning technologies shape student support curricula. With experience leading assessment initiatives, she is eager to explore strategies that provide targeted support for first-year success.

Abstract

In 2023, an AI-driven learning, tutoring and assessment programme, ALEKS, was integrated into the teaching and learning of mathematics, alongside developing a student support system informed by National Benchmark Test (NBT) data. ALEKS is a digital solution system founded by Dr Jean-Claude Falmagne and owned by McGraw-Hill (ALEKS, 2023). This platform was selected for piloting a solution to support incoming first-year students in their mathematics courses.

1. Introduction

In the initial stages of university life, students often grapple with the accelerated pace of learning, increased independence and the demand for critical thinking. These struggles can affect student retention, attrition rates and overall engagement with the learning process. Student academic profiles differ; even within the same institution and course, their learning experiences are not the same. Despite the wealth of valuable data on student performance, the manual analysis and implementation of support programmes remain time-consuming and resource-intensive for first-year lecturers, who are already burdened with heavy teaching responsibilities. Recognising this, the need for a smarter, more efficient solution became evident.

2. Solution

Given the blended learning goal of the DMISRS project, the main focus in 2023 was finding a learning platform to help students connect their school mathematics knowledge and the university courses in which they are enrolled, and to bridge potential transitional gaps to allow for greater success at university. The platform selected for piloting this concept was ALEKS – Assessment and LEarning in Knowledge Spaces. ALEKS seemed well-suited to the purpose because it is an established and widely used platform, easily modifiable, and has an adaptive learning element. A drawback of the platform is that the content is developed for the US context, so care needs to be taken in selecting content to ensure relevance. ALEKS uses knowledge space theory to determine what an individual student knows and what the student is ready to learn next. Knowledge space theory emerged from ground-breaking research in mathematical cognitive science. Dr Jean-Claude Falmagne, the founder of ALEKS, was instrumental in this research.

The NBT ALEKS solution used NBT data and relative importance analysis (RIA) to start designing course-specific intervention programmes that run alongside existing classes, without adding to lecturers' workloads. The idea was to use RIA for the historical course-level performance indicators and NBT mathematics subdomains to design a "bridging" course

curriculum. The adaptive learning platform then takes care of the differences in students' profiles, making it possible for students to benefit from the targeted course uniquely. This concept is illustrated in Figure 1 below.

Although the programme was introduced in mid-2023, missing the ideal first six weeks of the transition from high school to university, it has set the stage for future improvements.

3. Reflections

We attempted to run multiple pilots across different institutions and with different groups of students (Naidoo, 2023). The main lesson learnt in implementing a multi-institutional support initiative is that a programme such as this is unlikely to succeed without the full commitment of the course convenors. Despite initial interest from faculties and course convenors, the programme was not fully implemented due to a lack of sustained commitment. However, students who participated gave us positive feedback, and there was early evidence of learning gains.

4. Observed change

Over 2023 and 2024, the pilot reached more than 1 000 students, with a participation rate of 84%. Students were willing to participate in the programme, understanding that mathematics performance could be the key to success. The additional impact goal that emerged during this project, which holds significant implications for future studies, is educators' keen interest in digital solutions to enhance student support programmes and their readiness to participate in pilot initiatives.

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Appendix K: The “spaghetti approach” to teaching mathematics

Case study author: Jonathan Shock

Prof Jonathan Shock (JS) is originally a theoretical physicist and now teaches in the Department of Mathematics and Applied Mathematics at UCT. For nearly a decade, he has convened the first-year “Maths for scientists” course, which serves around 500 students annually. His focus is on teaching large, diverse classes effectively. Recently, he contributed to developing a mathematical thinking course to support at-risk first-year science students. His teaching philosophy involves experimenting with small interventions to see what works – a method that he calls the “spaghetti approach”.

Abstract

There are about 500 students in my course, and they have lectures five days a week throughout the year, alongside a weekly two-hour tutorial. The cohort is incredibly diverse, with incoming students having vastly different high-school experiences and thus different mathematical backgrounds. Over the last decade, I have experimented with many different small interventions to try to engage different students who are either struggling or finding the pace too slow. Some of these interventions have been very effective, while others have not hit the mark.

1. Introduction

Having such a diverse group of students in one class raises interesting challenges and opportunities. The fundamental question is: Given a fixed curriculum to get through, how do we keep everyone engaged in and outside of the classroom? The fast pace of the classes means that students can easily fall behind, and when they do, it is very difficult for them to catch up. It is important to ensure this does not happen in the first place. As there are many competing demands on students’ time and attention, this is a very difficult challenge, so I have implemented many different approaches over the years to engage as many students as possible.

2. Solution

Some of the interventions that I have implemented are:

- Giving students red cards that they can show me indicates that I am not explaining things well enough. This technique takes the pressure off them and shows that if they do not understand something, it may well be the teacher who is at fault.
- Creating a blogging platform (www.mathemafrika.org) where I blog about mathematics lectures, and students can also write posts to share their insights and experiences with other students.
- Giving prizes for the most improved students between tests helps students who do badly in the first tests, can win prizes by improving, rather than by having to come first in the class.
- Implementing peer-marking for voluntary assignments where students mark each other’s work.

- Implementing mathematical thinking workshops for at-risk students.
- Hiring an in-house psychologist to assist with student mental-health issues, time-management skills and general mathematics anxiety.
- Hiring second-year students to be tutors for the first years, as second years have the most recent perspective on what it is like to be a first year.

3. Reflections

A major impact of the interventions has been students seeing that the lecturer cares about their learning, positively affecting their desire to be a part of the course. Some of the interventions, like the red-card system, tend to be met with excitement initially, and then students forget about them relatively quickly. Other interventions, like the mathematical thinking workshops, seem to have a longer-term impact.

4. Observed change

Given the relatively high failure rates, the course was originally seen as a course impeding graduation. The pass rate over the last few years has improved significantly, although it is always very hard to tell exactly the reason for such an improvement in such an uncontrolled setting.

Appendix L: Beyond numbers: Exploring the role of academic literacy in mathematics performance

Case study authors: Sanet Steyn and Tatiana Sango

Ms Sanet Steyn (SS) is an applied linguist specialising in language assessment, focusing on entry-level academic literacy skills. Her interest in the relationship between academic literacy and mathematics performance emerged through the DMISRS project, leading to her involvement in various research initiatives. One such project examined cross-domain relationships between academic literacy and mathematics, exploring how performance in these areas is connected among prospective first-year university students.

Abstract

The National Benchmark Tests (NBTs) provide us with rich diagnostic data that can be used to augment our understanding of a student's profile or a cohort's needs. However, our approach to using this information must be undergirded by empirical evidence and a clear understanding of the relationships between the domains that we assess in the NBTs and their relevance to the disciplines taught at the higher education level. It is therefore essential to also explore how subject-specific and cross-domain skills (in this case, those associated with mathematics and academic literacy) are acquired and the cognitive demands involved in acquiring these skills. Using a sample cohort, this study explored a variety of potential analyses of the relationships between students' performance in these domains to identify some useful elements to add to our existing reporting and determine which relationships need further investigation.

1. Introduction

Some of the most pervasive questions that the Centre for Educational Assessments faces in our work with higher education institutions focus on the relevance of the domains assessed in the NBTs to the various disciplines taught at university. Therefore, how a student's academic literacy level might impact their performance in a non-language subject such as mathematics has been raised many times. Moreover, to enhance curriculum responsiveness, it has become very important for us to provide resources to help students and lecturers interpret the diagnostic results of tests such as the NBT Academic Literacy and Quantitative Literacy (NBT AQL) test.

With this in mind, we engaged in some qualitative investigations to describe the potential relationship between mathematics and academic literacy; this signalled the need for a deep dive into the performance data of the NBT Mathematics test and the Academic Literacy component of the NBT AQL.

In essence, two main questions need to be answered: (1) What is the underlying relationship between performance in the Mathematics (MAT) and Academic Literacy (AL) tests among prospective first-year university students, and (2) do specific subdomains or cognitive levels contribute significantly to this relationship?

We selected a sample for our case study to do some preliminary analysis and explore potential methodologies for investigating these relationships.

2. Solution

We explored:

- the connection between students' scores in the MAT and AL tests;

- the patterns or shared factors that explain the correlation between performance in the two subjects;
- Whether certain subdomains (e.g. algebraic processing, inferencing) or cognitive levels of items (e.g. basic application, higher-order reasoning) play a role in shaping the relationship between MAT and AL scores.

This research examined how mathematics and academic literacy are interconnected, advocating for ability grouping in classrooms to provide differentiated instruction and support based on the skills from both areas. In the early phases of this research, Item Response Theory (IRT) methods and relative importance analysis (RIA) were instrumental in shaping hypotheses, refining research inquiries and justifying the subsequent steps in the study all feeding into a broader research initiative, with upcoming procedures such as model fitting, validation and more detailed analyses planned for, among other things, a cognitive diagnostic modelling (CDM) investigation and a project looking at reconceptualising our existing reporting mechanisms to enhance their utility by incorporating data points that draw on this work.

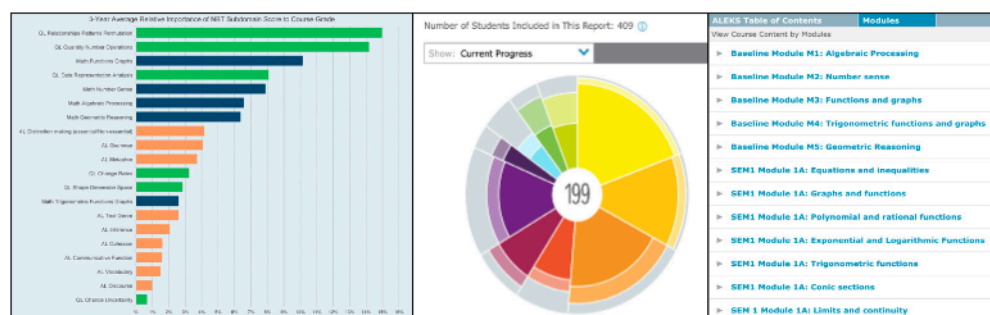


Figure 11: From relative importance analysis (RIA) to NBT ALEKS curriculum

3. Reflections

Key insights that will guide future efforts include:

- The clustering of correlations among MAT and AL overall scores and theta (modelled) scores.
- RIA of MAT and AL subdomains reveals valuable information through deeper exploration, including the breakdown of MAT subdomains into finer subskills.
- Work on the Q-matrix (borrowed from CDM) indicates trends regarding the influence of AL subdomains, aligning with RIA findings.
- Assessing performance across cognitive levels in MAT and AL offers vital information for qualitative mapping and informing us of the relationships within and between the domains assessed in the NBTs.

4. Observed change

This case study reinforced the need for a more nuanced approach to reporting student performance by incorporating mathematics and academic literacy insights. The findings have directly informed the broader research initiative aimed at reconceptualising existing reporting mechanisms to enhance their utility, ensuring that diagnostic data is leveraged more effectively to support students and lecturers.