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THE SIMILARITY LAWS FOR MEMS GYRO WITH TEMPERATURE CHANGES

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The performance of MEMS gyro is affected by several factors, and one of them is the change of temperature. To design a higher performance MEMS gyro, the temperature compensation should be taken into account. Due to its complicated nature, the temperature data has to been collected via experiments. Dimensional analysis should was used to find the similarity law of temperature effects on resonant frequency, Q factor and voltage output. Since there were no restrictions on the MEMS gyro structure, it appears that all the similarity laws may be valid to any other mechanical or electro-mechanical system.

1. Introduction

Because of the rapid development of the Internet of Things (IoT), motion sensors such as MEMS gyros have become increasingly required. However, the performance of the MEMS gyro is negatively affected by a change in temperature, and in general, the MEMS gyro cannot be used for high-end field without temperature compensation. In practice, the temperature compensation of each MEMS gyro should be tested one-by-one, which is costly and time consuming. various research investigated the temperature problem of the MEMS gyro, and have reached certain findings for each particular case[1,2,3,4]. However, owing to the complicated nature of the problem, there is no result on the general consideration on the problem. This paper reports on an investigation of the problem by using dimensional analysis. Some similarity laws have been formulated. Those general results must benefit the future development of a high performance MEMS gyro.

2. Dimensional analysis and Buckingham Pi theorem[5,6]

In many cases in real-life engineering, the equations are either not known or too difficult to solve; often experimentation is the only method of obtaining reliable information. In most experiments, in order to save time and money, tests are performed on a geometrically scaled model, rather than on the full-scale prototype. In such cases, care must be taken to properly scale the results.

All mathematical equations must be dimensionally homogeneous; this fundamental principle can be applied to equations in order to nondimensionalize them, and to identify dimensionless groups, which are also called non-dimensional parameters. A powerful tool to reduce the number of necessary independent parameters in a problem is called dimensional analysis. Dimensional analysis is useful in all disciplines, especially when it is necessary to design and conduct experiments.The beauty of dimensional analysis is that the only other thing that we need to know is the primary dimensions of each of these quantities.

To generate the non-dimensional parameters, namely, the $\Pi's$. there are several methods that have been developed for this purpose, but the most popular (and simplest) method is the method of repeating variables, which was popularized by Edgar Buckingham.The method of repeating variables is a step-by-step procedure to find the non-dimensional parameters, or $\Pi's$, based simply on the dimensions of the variables and constants in the problem.

Buckingham Pi theorem: The uppercase Greek letter Π denotes a non-dimensional parameter. In a general dimensional analysis problem, there is one Π that we call the dependent Π, giving it the notation Π_1. The parameter Π_1 is in general a function of several other Π, which we call independent Π The functional relationship is $\Pi_1 = f(\Pi_2, \Pi_3, ..., \Pi_k)$, where k is the total number of Π.

Consider an experiment in which a scale model is tested to simulate a prototype flow. To ensure complete similarity between the model and the prototype, each independent $\prod$ of the model (subscript m) must be identical to the corresponding independent Π of the prototype (subscript p),namely,$\Pi_{2,m} = \Pi_{2,p}, \Pi_{3,m} = \Pi_{3,p}, ..., \Pi_{k,m} = \Pi_{k,p}$, where m and p denote model and prototype.

Under these conditions the dependent Π of the model $(\Pi_{1,m})$ is guaranteed to also equal the dependent Π of the prototype $\Pi_{1,p}$. Mathematically, we write a conditional statement for achieving similarity, if $\Pi_{2,m} = \Pi_{2,p}, \Pi_{3,m} = \Pi_{3,p}, ..., \Pi_{k,m} = \Pi_{k,p}$, then $\Pi_{1,m} = \Pi_{1,p}$.

The six steps in the method of repeating variables are summarized below:

Step 1: List the parameters in the problem and count their total number n. List the parameters (dimensional variables, non-dimensional variables, and dimensional constants) and count them. Let n be the total number of parameters in the problem, including the dependent variable. Make sure that any listed independent parameter is indeed independent of the others, which means that, it cannot be expressed in terms of them.

Step 2: List the primary dimensions for each of the n parameters.

Step 3: Guess the reduction j. As a first guess, set j equal to the number of primary dimensions represented in the problem. The expected number of $\Pi'(k)$ is equal to n minus j, according to the Buckingham Pi theorem. If at this step or during any subsequent step, the analysis does not work, verify that you have included enough parameters in step 1. Otherwise, return and reduce j by one, and try again.

Step 4: Choose j repeating parameters that will be used to construct each Π. Since the repeating parameters have the potential to appear in each Π, one should choose them wisely.

Step 5: Generate the $\Pi's$ one at a time by grouping the j repeating parameters with one of the remaining parameters, forcing the product to be dimensionless. In this way, construct all k $\Pi's$. By convention the first Π, which is designated as Π, is the dependent Π_1. Manipulate the $\Pi's$ as requires to achieve established dimensionless groups.

Step 6: Write the final functional relationship and check your algebra.

3. List of common physical quantities and their dimensions that are used in MEMS gyro

There is a difference between dimensions and units: a dimension is a measure of a physical quantity (without numerical values), while a unit is a way to assign a number to that dimension. There are seven primary dimensions in all fields of science and engineering, which are mass, length, time, temperature, electric current, amount of light, and amount of matter. All other dimensions can be formed by a combination of these seven primary dimensions.

The primary dimensions of each parameter are listed below along with each dimension with exponents, since this helps with later algebra.

Table 1. List of common physical quantities and dimensions

Quantity	Symbol	Dimension
Mass	M	M
Length	L	L
Time	t	t
Temperature	T	Θ
Electric current	I	$A(ampere)$
Velocity	v	Lt^{-1}
Frequency	f	t^{-1}
Angular-frequency	ω	t^{-1}
Density	ρ	ML^{-3}
Viscosity	μ	$ML^{-1}t^{-1}$
Kinematic viscosity	ν	L^2t^{-1}
Capacitance	C	$L^{-2}M^{-1}t^4I^2$
Resistance	R	$L^2Mt^{-3}I^{-2}$
Inductance	L	$L^2Mt^{-2}I^{-2}$
Charge	q	tI
Voltage	V	$L^2Mt^{-3}I^{-1}$
Damping coefficient	c	Mt^{-1}
Young modulus	E	$L^{-1}Mt^{-2}$
Free path distance	λ	L
Knudsen number	$K_n = \frac{\lambda}{L}$	1
Reynolds number	$R_e = \frac{\rho L v}{\mu}$	1
Q factor	$Q = \frac{\sqrt{MK}}{c}$	1
Gyro angular-freq.	Ω	t^{-1}

4. The similarity law for natural frequency for MEMS structure with temperature changes

From the theory of vibration we know that the natural angular frequency of the MEMS structure (or any structure) can be expressed as $\omega = 2\pi f = \sqrt{\frac{K}{M}}$, where K and M are the global stiffness and the total mass of the structures, respectively. When the structure undergoes temperature change, the total mass remains and length and Young modulus of the structure will change, which will result in a change in the global stiffness of the structure. Finally, the natural frequency of the structure will change accordingly.

The architecture of MEMS gyro structure has numerous combinations, hence it is impossible to verify each of them. What we like to do is find the similarity law for an arbitrary structure of MEMS gyro with the help of dimensional analysis.

SOLUTION: We should to generate a non-dimensional relationship between natural frequency and other parameters, including temperature.

Assumptions: 1. Isotropic elastic material. 2. Temperature gradient is small. 3. Length change is linear proportional to temperature change. 4. Young modulus change is linear and proportional to temperature change. 5. No other parameters are significant in the problem.

Analysis: The method of repeating variables is employed to obtain the non-dimensional parameters.

Step 1: There are five variables and constants in this problem; $n = 5$. They are listed in functional form, with the dependent variable listed as a function of the independent variables and constants: $\Delta f = y(K, M, \alpha, \Delta T), \Delta f = f - f_0, \Delta T = T - T_0$.

Step 2: The primary dimensions of each parameter are listed below.

$\triangle f$	K_0	M	α	$\triangle T$
t^{-1}	Mt^{-2}	M	Θ^{-1}	Θ

Step 3: Firstly, j is set equal to 3, the number of primary dimensions represented in the problem (M, t, and K).

Step 4: We choose three repeating parameters since $j = 3$.The best choice of repeating parameters is M, t, T.

Step 5 The dependent Π is generated as follows:

$\Pi_1 = \Delta f K_0^a M^b (\triangle T)^c$

or in dimensional format, $\Pi_1 = t^{-1}(Mt^{-2})^a (M)^b (\Theta)^c$

from which $a = -\frac{1}{2}$, $b = \frac{1}{2}$ and $c = 0$, and thus the dependent Π is $\Pi_1 = \Delta f K_0^{-\frac{1}{2}} M^{\frac{1}{2}}$.

Similarly, the two independent $\Pi's$ are generated, the details of which are

$\Pi_2 = \alpha K_0^{a_1} M^{b_1} (\Delta T)^{c_1}$

from which $a_1 = 0, b_1 = 0, c_1 = 1$, and hence

$\Pi_2 = \alpha \Delta T$.

Step 6: We write the final functional relationship as:

$\Delta f \sqrt{\frac{M}{K_0}} = y(\alpha \Delta T)$ or $\Delta f = \sqrt{\frac{K_0}{M}} y(\alpha \Delta T)$ in which the function y can be obtained via tests. In some published tests the y can be determined as a linear function, and then we have the similarity law of the frequency vs. temperature as $\Delta f = -\frac{1}{2\pi}\sqrt{\frac{K_0}{M}} \alpha \Delta T$.

To verify the similarity law[4], let K_0be the stiffness of the structure at the absolute temperature T_0, then we have stiffness at temperature T, $K = K_0[1-\alpha(T-T_0)]$, or rewritten as $\triangle K = -K_0 \alpha \triangle T$, and $\triangle K = K - K_0, \triangle T = T - T_0$. Assuming that $K = K_0(1 - \alpha \Delta T)$, from that definition of the natural frequency, we have $f = \frac{1}{2\pi}\sqrt{\frac{K}{M}} = \frac{1}{2\pi}\sqrt{\frac{K_0(1-\alpha\Delta T)}{M}} \simeq \frac{1}{2\pi}\sqrt{\frac{K_0}{M}}(1 - \alpha \Delta T)$, which is $\Delta f = -\frac{1}{2\pi}\sqrt{\frac{K_0}{M}} \alpha \Delta T$.

If we ignore the coefficient in the similarity law, then we have simplest similarity law as

$$\Delta f \sim -\Delta T. \tag{1}$$

which simply states the law of similarity as follows: the natural frequency change is negatively linear proportional to the change of temperature.

5. The similarity law for Q-factor for an MEMS driving structure with temperature changes

In the design of Z axis gyro, the architecture is in plane motion in which both driving and sensing modes are in plane of the gyro structure. Generally, the driving uses a sliding mode and sensing adopts squeezing mode. The viscosity and friction for the sliding and squeezing mode is different, we study the problem separately.

For the problem we can list all parameters and their dimensions as:

$D - mass^*$	$resis - coef$	$stiffness$	$force - amp$	$force - freq$	α	$Temp$
M_D	C_D	K_D	F_D	ω_F	α	ΔT
M	Mt^{-1}	Mt^{-2}	LMt^{-2}	t^{-1}	K^{-1}	K

* D means Driving

We have n=7 parameters and j=4 repeating parameters. $M_D, K_D, F_D, \Delta T$ are the repeating parameters, and c_D, ω_D, α are the n-j=3 dependent variables.

From the dimensional analysis we have three dependentΠ as $\Pi_1 = c_D M_D^{-\frac{1}{2}} K_D^{\frac{1}{2}}$, $\Pi_2 = \omega_D M_D^{\frac{1}{2}} K_D^{-\frac{1}{2}}$, $\Pi_3 = \alpha \Delta T$. Then we have the similarity law as $c_D M_D^{-\frac{1}{2}} K_D^{\frac{1}{2}} = y(\omega_D M_D^{\frac{1}{2}} K_D^{-\frac{1}{2}}, \alpha \Delta T)$, which can be rewritten in a more compact format as $\frac{1}{Q_D} = y_D(\frac{\omega_F}{\omega_D}, \alpha\Delta T)$ in which the driving mode Q factor is $Q_D = \frac{\sqrt{M_D K_D}}{c_D}$, and the resonant frequency of the driving structure is $\omega_D = \sqrt{\frac{K_D}{M_D}}$.

If the driving frequency is equal to the resonate frequency of the driving structure, namely, $\omega_F = \omega_D = \sqrt{\frac{K_D}{M_D}}$, then we have the simpler similarity law as $\frac{1}{Q_D} = y(\alpha\Delta T)$, and furthermore, from tests, the function y is approximately negative linear function, so we have approximate similarity law as $Q_D \sim -\frac{1}{\sqrt{\Delta T}}$.

In reality, if the temperature changes, the natural frequency of the driving structure will be shifted to $\omega_F \neq \omega_D$, even it equals the driving force frequency in the beginning. Generally,if we linearized the similarity law then we have $\frac{1}{Q_D} = -\alpha\sqrt{\Delta T}$. It states the scaling exponent is "$-\frac{1}{2}$" , then we have

$$Q_D = -\frac{1}{\sqrt{\Delta T}}. \tag{2}$$

6. The similarity law for the Q-factor for the MEMS sensing structure with temperature changes

Similarly, we can get the Q factor for the sensing mode as $\frac{1}{Q_S} = y_S(\frac{\omega_G}{\omega_S}, \alpha\Delta T)$, in which G and S represents G-force and Sensing, hence the resonant frequency of the sensing structure is $\omega_S = \sqrt{\frac{K_S}{M_S}}$. The first order approximation is $Q_S \sim -\frac{1}{\sqrt{\Delta T}}$, which follows the similarity law of "$-\frac{1}{2}$" on the change of temperature.

$$Q_S \sim -\frac{1}{\sqrt{\Delta T}}. \tag{3}$$

7. The similarity law for Q-factor of the MEMS whole structure with temperature changes

For a combination of the sensing and driving structure, we have a total structure of Q_W factor as $\frac{1}{Q_W} = \frac{1}{Q_D} + \frac{1}{Q_S}$, or $\frac{1}{Q_W} = y_D(\frac{\omega_D}{\sqrt{\frac{K_D}{M_D}}}, \alpha\sqrt{\Delta T}) + y_S(\frac{\omega_S}{\sqrt{\frac{K_S}{M_S}}}, \alpha\sqrt{\Delta T}) \approx -\beta_D\sqrt{\Delta T} - \beta_S\sqrt{\Delta T} \sim -\beta\sqrt{\Delta T}$, which means that the law of similarity for the MEMS gyro structure indicates that the good approximation of the total Q factor is negatively linear proportional to the temperature change, namely, $Q_W \sim -\frac{1}{\sqrt{\Delta T}}$. It follows the similarity laws of "$-\frac{1}{2}$" on the change of temperature.

$$Q_W \sim -\frac{1}{\sqrt{\Delta T}}. \tag{4}$$

8. The similarity law of voltage output for MEMS gyro with open loop with temperature changes

MEMS gyro is an electro-mechanical system, which consists of a mechanical part and an electrical part, hence we will consider all aspects of the system and discover the MEMS gyro similarity law.

For mechanical subsystem, the sensing mode is responsible for collecting signal and input to the electrical subsystem, and for analysis we only need to list all the important parameters for the sensing mode as follows: mass M_S, resistance coefficient c_S, stiffness K_S, external G-force $F = 2\Omega v$, G-force mode angular frequency ω_G, thermal coefficient α, and temperature change ΔT. For the electrical subsystem, we have the following parameters: resistance R, capacitance C, inductance L and voltage output V. In total, we have 7 thermal-mechanical parameters and 4 electrical parameters.

The total number of parameters is n=11. There are j=5 primary parameters such as length, mass, time, current and Kelvin. We can choose $M, K, F, \Delta T, R$ as repeating parameters and will need to generate 6 dependent $\Pi's$.

From the dimensional analysis, we achieved at the 6 dependent $\Pi's$ as follows: $\Pi_1 = c_S M_S^{-\frac{1}{2}} K_S^{-\frac{1}{2}}$, $\Pi_2 = \omega_G M_S^{\frac{1}{2}} K_S^{-\frac{1}{2}}$, $\Pi_3 = \alpha \Delta T$, $\Pi_4 = V Q_S^{\frac{1}{4}} c_S^{\frac{1}{4}} (\Omega v)^{-1} R^{-\frac{1}{2}}$, $\Pi_5 = C M_S^{-\frac{1}{2}} K_S^{\frac{1}{2}}$,$\Pi_6 = L Q_S^{-1} c_S^{-1} R^{-1}$. If we introduce the electrical resonant angular frequency ω_E, then we have the Q-factor for the electrical subsystem $Q_E = \frac{\omega_E L}{R}$, then some of the above $\Pi's$ can be reformatted as follows $\Pi_1 = Q_S^{-1}$, $\Pi_2 = \omega_G \omega_S^{-1}$, $\Pi_3 = \alpha \Delta T$, $\Pi_4 = V Q_S^{\frac{1}{4}} c_S^{\frac{1}{4}} (\Omega v)^{-1} R^{-\frac{1}{2}}$, $\Pi_5 = C \omega_S$,$\Pi_6 = Q_E Q_S^{-1} \omega_E c_S^{-1}$,where the resonant frequency of sensing structure $\omega_S = M_S^{\frac{1}{2}} K_S^{-\frac{1}{2}}$. The we will have the voltage output for the open loop: $V = Q_S^{-\frac{1}{4}} c_S^{-\frac{1}{4}} \Omega v R^{\frac{1}{2}} V(Q_S^{-1}, \omega_G \omega_S^{-1}, C\omega_S, Q_E \omega_E Q_S^{-1} c_S^{-1}, \alpha \Delta T)$.

Assuming that the voltage is the linear function of the temperature change, then we have the simplest relation as $V = Q_S^{-\frac{1}{4}} c_S^{-\frac{1}{4}} R^{\frac{1}{2}} \alpha \Omega v \Delta T$. Since $Q_S \sim \frac{1}{\sqrt{\Delta T}}$, the fiction coefficient is $c_s \sim \frac{1}{\sqrt{\Delta T}}$ and the electrical resistance $R \sim \Delta T$. Then the similarity laws become $V \approx \alpha \Omega v \Delta T^{\frac{7}{4}}$, which means that the sensing scaling exponent is the "$\frac{7}{4}$" , then we have

$$V \approx \alpha \Omega v (\Delta T)^{\frac{7}{4}}. \tag{5}$$

9. Conclusion and future research

The performance of the MEMS gyro is affected by several factors, and one of them is a change in temperature. To design a high performance MEMS gyro, the temperature compensation should be taken into account. Hence, owing to its complicated nature, the temperature data was collected via experiments. The dimensional analysis was used to find the similarity law of temperature effects on the resonant frequency, Qfactor and voltage output.

Some similarity laws or scaling laws can be summarized as follows:

1. The natural frequency change is negatively linear and proportional to the change of temperature;

2. Q-factor of sensing,driving and the whole MEMS structure follows the scaling laws of "$-\frac{1}{2}$" regarding the change of temperature;

3.The similarity law of the sensing voltage output of MEMS gyro with open loop has the scaling exponent as "$\frac{7}{4}$" regarding the change of temperature, and

4. Since there were no restrictions on the MEMS gyro structure, it seems that all our similarity laws may be valid for any other mechanical or electro-mechanical system.

It should be pointed out that the temperature effects on the MEMS gyro is complicated and the above discussion is a primary attempt to investigate the problem, which requires further research. The key to the problem is how to find the resistance or fiction coefficient of MEMS gyro for different kinds of motion, such as sliding and squeezing. For a chips level vacuum packaging, the fiction analysis must take into account rarefied gas dynamics. The high velocity vibration of the driving mode forces the gas in the chips cavity into turbulence in a small Reynolds number (is about 1). Hence, the turbulence fiction analysis of the MEMS structure in the rarefied gas environment with small Reynolds number and a large Knudsen number should be considered in future research.

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